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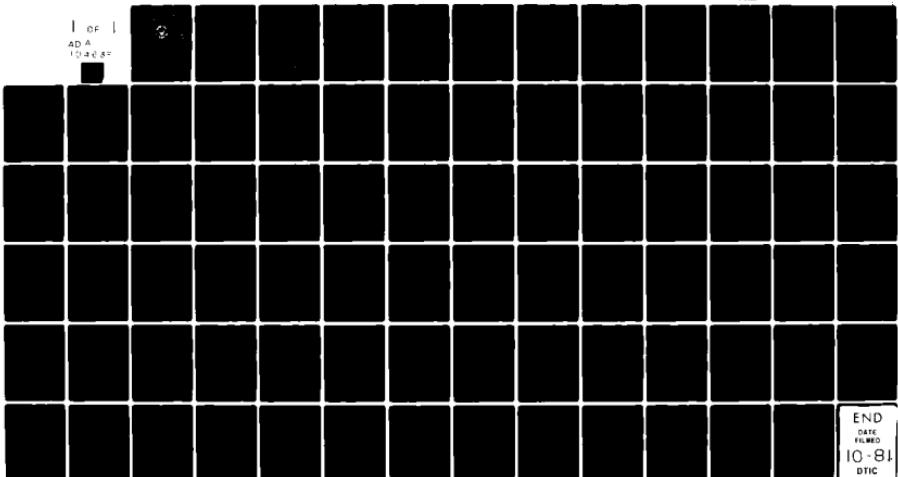
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THESIS

A SIMULATION ANALYSIS OF THE EFFECTIVENESS  
OF MARKOVIAN CONTROL AND BAYESIAN CONTROL

by

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11 Jun 1981

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(20. ABSTRACT Continued)

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A Simulation Analysis of the Effectiveness  
of Markovian Control and Bayesian Control

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### ABSTRACT

Statistical cost control decisions may be based on two competing decision models. The Markovian Control model controls a process by investigating the process whenever the reported cost exceeds a fixed critical limit. The Bayesian Control model controls a process by using the reported cost to update the probability of the process being in-control and investigate the process whenever such posterior probability is less than a fixed critical value.

This paper compares the relative effectiveness of the two models by a simulation analysis. It is observed that the Markovian Control model performs as well as or better than the Bayesian Control model unless the cost distribution of the in-control state is more dispersed than that of the out-of-control state. It is also observed that the relative effectiveness of the Markovian Control model compared to the Bayesian Control increases as the savings from an investigation increases when the cost distribution of the in-control state is less dispersed than that of the out-of-control state.

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## I. INTRODUCTION

### A. BACKGROUND

Financial planning and control rely heavily on standard costs or budgets as the principal tool for aiding decision making. Performance reports typically include a comparison of actual cost incurred and amount budgeted or allowed. Variations in cost performance can be expected in virtually every case. The variation may be caused by random variation or non-random variation. Random variation is a deviation between actual and expected cost arising from the stochastic operation of some correctly specified relationships between variables. The stochastic nature means that the actual cost is subject to fluctuations because of random variations beyond management's control. Non-random variation is a deviation in cost performance caused by factors over which the manager or his superior has some control.

Non-random variation can be corrected by investigating the cost generating process to identify the causal factors for corrective decisions. But no one advocates taking action and investigating every cost variance that occurs each period. Managers recognize that many variances are insignificant and caused by random, non-controllable factors. Since any investigation will involve a certain expenditure of effort and funds, managers should attempt to take investigation and corrective action only when the cost variation is significant.

and controllable. Furthermore an investigation should be undertaken only if the benefits expected from the investigation exceed the costs of searching for and correcting the source of cost variance.

Therefore, a control problem in the cost management is to establish a control policy according to which a cost generating process shall be investigated to see if corrective action is necessary or economic.

#### B. ISSUES IN COST VARIANCE INVESTIGATION DECISIONS

Several researchers have attempted to develop cost variance investigation decision models by using a statistical approach. The basic idea of these approaches came from the statistical quality control concept in which it is assumed that some random variations in cost performance are available and should be specifically taken into consideration in making decisions about investigation of variances.

Since a manager wants to control the cost generating process, he should decide when a variance is worthy of investigation. If the variance resulted from non-controllable factors, or if future operations would not improve even if the cause of the variance was determined, he would prefer not to waste time and money investigating such variances. On the other hand, if investigation will result in substantial future savings and more efficient operation, he will probably want the variance investigated. For deciding whether to investigate the process or not, the manager's available information consists

of some prior knowledge about the process and a cost report which was generated from the process. Therefore, in deciding whether to investigate a variance of the cost report, the following factors should be considered [1].

- a. The probability that the variance resulted from the random, non-controllable causes.
- b. The reward which will result if the variance is investigated together with the associated probability of this reward.
- c. Cost of investigation.

It is presumed that the costs generated from the process have some kind of probability distribution regardless of the cause of the deviations and that different causes will result in different probability distributions in cost performance. Attributing to the characteristics of probability, a decision maker always associates some probability of errors (which are called type I and type II error) with his judgment about whether a cost variance is caused by random factors. If the distributions of costs which may be caused by controllable or non-controllable factors are normal distributions, these errors can be explained as in Figure 1-1.

In Figure 1-1, we can say that the left side probability distribution function (p.d.f.) is the p.d.f. of costs in "in-control" state, a desirable state, and the right side p.d.f. is the p.d.f. of costs in "out-of-control" state, an undesirable state which needs investigation. We assume that the process under consideration exists in either in-control state or out-of-control state. The type I error means that the

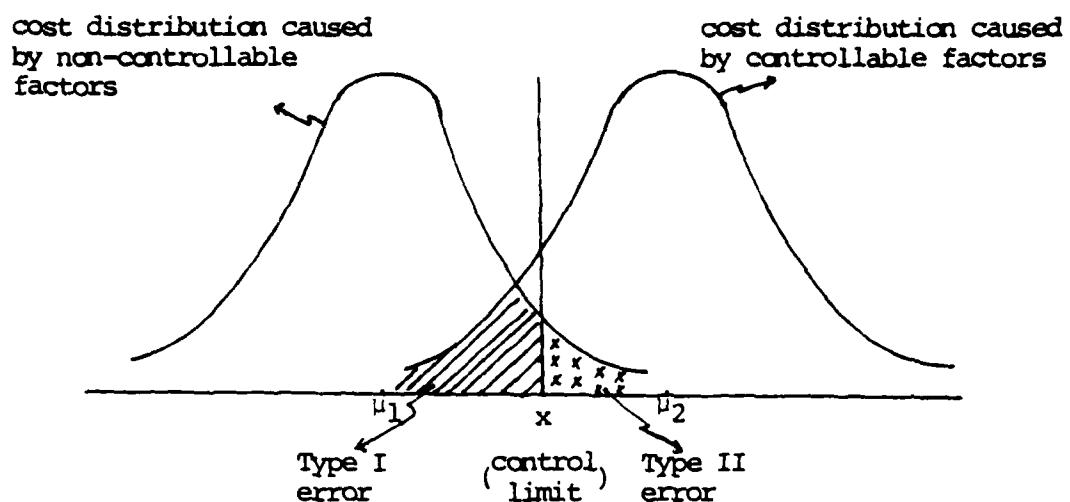


Figure 1-1: Error of Decision

manager decides not to investigate the process when the actual state is out-of-control; thus it causes loss of potential savings from investigation. The type II error means that the manager decides to investigate the process when the actual state is in-control; thus the cost of investigation is wasted.

Therefore, the manager has to figure out what control limit or control criteria (in the figure the control limit  $x$ ) can minimize those costs (loss and waste). All statistical approaches to cost control decisions are aimed at finding these control criteria.

The statistical control methods can be classified into two types: (1) the Markovian control method and (2) Bayesian control method. Both types suggested setting control limits on which the manager decides whether or not to investigate

the process when a cost report from the process is available. The underlying objective in both cases is to minimize long run expected incremental cost (waste and loss) or maximize savings from investigation (cost reduction minus incurred cost for the investigation).

The Markovian Control method was suggested in the article "Cost Variance Investigation; Markovian Control of Markov Process" [2], by Dittman and Prakash in 1978. Under this method, the cost variance investigation decision is dependent on one critical limit which minimizes the cost of the process and can be computed by trial and error.

The Bayesian control was suggested in the article "The Investigation of Cost Variance" [3], by Dykman in 1969. Under this method, the cost variance investigation decision is determined by the critical probability which is a function of all incremental costs involved.

The two statistical approaches found in the literature do not always result in the same investigate/do-not-investigate decision. A question arises as to which model will lead to the optimal decisions. Using an analytic approach, Dittman and Prakash maintained that their Markovian control model is more effective than the Bayesian control model. However, the real test of the relative effectiveness of each model under different conditions remained to be seen. As Kaplan [4] puts it, "the final judgment on the appropriateness of formal statistical and mathematical methods for cost variance analysis must be based on empirical studies" [4: p. 312]. In an empirical

study, Magee [5] implemented a simulation analysis of various cost variance investigation methods, but he didn't study the Markovian control method for it was not yet known. He justified simulation by saying that "the simulation analysis is generally preferable to use analytic methods to find the properties of alternative decision models, when such methods are feasible. By using the same sequence of random numbers, the various cost investigation models can be tested on similar cost data, facilitating comparisons among models" [5: p. 532].

Simulation is a dynamic representation of the real world achieved by building a model and moving it through time. In a simulation, we can control many features. For comparison of two cost variance investigation methods, we can simulate various combination of situations in the in-control state and out-of-control state. Simple Monte Carlo simulation can be a useful tool for comparison of two cost variance investigation methods.

#### C. OBJECTIVE AND SCOPE OF THE STUDY

The purposes of this paper are to develop a simulation model for the purpose of comparing the relative effectiveness of the two cost variance investigation methods and to evaluate the effectiveness of the two cost variance investigation methods.

To compare the best Markovian control with the best Bayesian control, the optimal critical limit for the Markovian control method and optimal critical probability for the Bayesian

control method must be derived. It is not too complicated to calculate the optimal critical limit, but the calculation of optimal critical probability is too complicated to be applied in the real world. A dynamic programming for calculating this critical probability was suggested by Kaplan [6]. But this calculation is beyond the scope of this paper. Instead of optimal critical probability, the breakeven probability, which is calculated from long-run expected savings and investigation cost, is used in this paper. Long-run expected savings may be estimated from the historical data. The estimation can be made by the method suggested by Duvall[8].

In this paper, it is assumed that the long-run expected savings were given as an external input to the simulation model.

The scope of this study is confined the comparison of the relative effectiveness of the two cost variance investigation methods. Whether or not a manager should use one of these two methods is a separate question and is not the subject of this study. There are several necessary assumptions underlying the two methods. These assumptions are not tested in this study.

#### D. METHODOLOGY

The approach taken in this paper to test the relative effectiveness of these two models is to (1) examine the procedures of each of the two methods by studying relevant articles, (2) simulate these procedures, (3) incorporate these procedures

into the simulation model, (4) transfer this model to computer program and run the program and (5) analyze the results.

This study is organized into five chapters; Chapter One presents the background and issues involved in cost variance investigation decisions, Chapter Two describes the detail of the two methods to be studied. Chapter Three presents the details of the simulation model, Chpater Four describes the results of the computer simulation for various cases, and finally, Chapter Five shows the conclusion derived from the analysis of Chapter Four data.

## II. DESCRIPTION OF COST VARIANCE INVESTIGATION METHODS

### A. MARKOVIAN CONTROL METHOD [2]

#### 1. Specification

The process operates in one of two possible conditions,  $i = 1, 2$ , of which condition 1 means that the process is in-control and condition 2 means that the process is out-of-control. If in control, the process may deteriorate into the out-of-control condition in the next period with a constant probability  $(1-g)$ . But, once out-of-control, the process continues to operate in that condition unless investigated and corrected. This is summarized in the following Markovian process "Transition Matrix":

$$\begin{matrix} & & 1 & 2 \\ & 1 & \begin{bmatrix} g & (1-g) \\ 0 & 1 \end{bmatrix} \\ 2 & & & \end{matrix} \quad (2-1)$$

The functioning of the process generates, in each period, say  $j$ , a cost  $X_j$ , which is a random variable, with its probability distribution function  $F_i(t)$ , depending upon the operating condition,  $i$ , of the process.

$$\Pr\{X_j \leq x | i\} = F_i(x) \quad i = 1, 2 \quad (2-2)$$

$$E(X | i) = \mu_i$$

The transition takes place before a cost is generated, so that cost reports provide information about the current status of the process.

On receiving a cost report, the manager faces a choice. A first alternative is to regard the process as having gone out-of-control, and so to incur a fixed discretionary investigation cost,  $I$ , which will reveal what, if anything, is wrong with the process. It is assumed that if the process is found to be out-of-control, it is reset to the in-control condition with a constant correction cost  $K$ , but if the process is found to be in-control, it is left to operate as is.

In the latter case, the manager is said to have committed an type I error (incurring a cost when it was not necessary).

The second alternative for the manager is to regard the process as being in-control, allowing it to run without intervention for one more period. In this case, the manager takes the risk of committing a type II error, that is, not investigating and correcting the process when, in fact, it was out-of-control.

## 2. Control Policy: A Single Critical Limit for the Cost Variance

Consider the class of control policies in which the manager bases his decision between "investigate" and "don't investigate" on whether the actual cost  $X$  exceeds some constant value  $x$ . Then the conditional probabilities of

committing type I and type II errors are constant and are, respectively, as follows:

$$\begin{aligned}\Pr\{\text{Investigate} | \text{in-control}\} &= \Pr\{X > x | i = 1\} \\ &= 1 - \Pr\{X \leq x | i = 1\} \quad (2-3) \\ &= 1 - F_1(x)\end{aligned}$$

$$\begin{aligned}\Pr\{\text{Don't investigate} | \text{out-of-control}\} &= \Pr\{X \leq x | i = 2\} \\ &= F_2(x) \quad (2-4)\end{aligned}$$

However, the unconditional probabilities of type I and type II errors do change with time, for they depend upon the probabilities of the process being in-control and out-of-control, which in turn, depend upon the number of periods elapsed since the last managerial intervention.

If the process is in-control when the report is produced, then, no matter what action the manager chooses, the process will start the next period in the in-control condition with probability 1. If, on the other hand, the process is out of control when the cost report is produced, then it will start the period in the out-of-control condition with probability equal to the probability of a type II error,  $F_2(x)$ , which is constant. Thus, the effect of manager's control action can be described by the following Markovian "Control Matrix":

$$\begin{array}{ccc}
 & 1 & 2 \leftarrow \text{condition of the} \\
 & 1 & \text{process after} \\
 1 & \left[ \begin{array}{cc} 1 & 0 \\ 1-F_2(x) & 1-F_2(x) \end{array} \right] & \text{"control" is} \\
 & 2 & \text{exercised} \quad (2-5) \\
 \uparrow & & \\
 \text{condition of the process when the cost} \\
 \text{report is produced}
 \end{array}$$

Therefore, we may define a new Markov process which combines the process Transition Matrix (2-1) and Control Matrix (2-5). We refer to it as the associated "Controlled (Markov) Process".

$$\begin{array}{ccc}
 \begin{bmatrix} g & (1-g) \\ 0 & 1 \end{bmatrix} & \times & \begin{bmatrix} 1 & 0 \\ 1-F_2(x) & F_2(x) \end{bmatrix} \\
 \text{Transition Matrix} & & \text{Control Matrix} \\
 = & \begin{bmatrix} 1-(1-g)F_2(x) & (1-g)F_2(x) \\ 1-F_2(x) & F_2(x) \end{bmatrix} & (2-6) \\
 & & \text{Controlled Process Transition Matrix}
 \end{array}$$

The steady-state probabilities  $\pi_i(x)$  ( $i = 1, 2$ ) of the controlled Markov Process (2-6) are given by:

$$\pi_2(x) = (1-g)\pi_1(x)F_2(x) + \pi_2(x)F_2(x) \quad (2-7)$$

$$\pi_1(x) + \pi_2(x) = 1$$

$$\pi_1(x) = \frac{1-F_2(x)}{1-gF_2(x)} ; \quad \pi_2(x) = \frac{(1-g)F_2(x)}{1-gF_2(x)} \quad (2-8)$$

As may be seen from Figure 2-1,  $\pi_1(x)$  and  $\pi_2(x)$  are the steady state probabilities for the states in which the process finds itself at the end of the managerial control (or, equivalently, at the start of a period of operation). They are not the same as the steady-state probabilities  $s_i(x)$  ( $i = 1, 2$ ) of the states generating the cost reports. Probabilities  $s_1(x)$  and  $s_2(x)$  are easily found by applying the process transition matrix (2-1) the the steady state vector  $\pi_1(x)$ ,  $\pi_2(x)$ :

$$s_1(x) = g_1(x)$$

(2-9)

$$s_2(x) = (1-g_1(x)) + g_2(x)$$

$$= \frac{g}{1 - gF_2(x)}$$

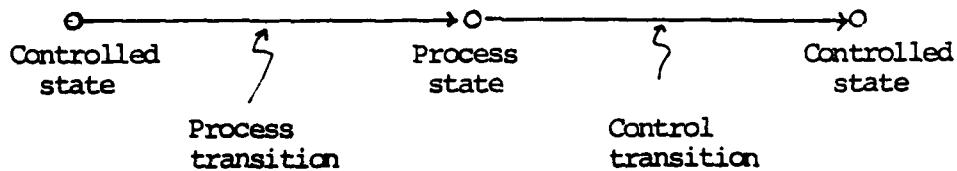


Figure 2-1: A Controlled (Markov) Process

### 3. The Expected Cost of Operating Controlled Process

The total expected cost per period is the sum of the expected cost per period of (i) operating, (ii) investigating, and (iii) correcting the process.

From Equations (2-2) and (2-9), the expected cost per period of operating the process equals:

$$\begin{aligned}
 C_o(x) &= \mu_1 s_1(x) + \mu_2 s_2(x) \\
 &= \mu_1 g \pi_1(x) + \mu_2 (1-g) \pi_1(x) + \mu_2 \pi_2(x) \\
 &= \mu_2 - \pi_1(x) g \Delta \mu
 \end{aligned}$$

where

$$\Delta \mu = \mu_2 - \mu_1$$

The investigation costs are incurred in the event that the cost report  $X > x$ . In the steady state, the unconditional probability of the event  $X > x$  is readily computed using Equations (2-3), (2-4), and (2-9).

$$\begin{aligned}
 \Pr\{X > x\} &= \Pr\{X > x | i = 1\} s_1(x) + \Pr\{X > x | i = 2\} s_2(x) \\
 &= 1 - F_1(x) g \pi_1(x) + \frac{(1-g) 1 - F_2(x)}{1 - g F_2(x)} \\
 &= \pi_1(x) \{1 - g F_1(x)\} \tag{2-11}
 \end{aligned}$$

Hence, the expected cost per period for investigating the process equals:

$$C_I(x) = \pi_1(x) \{1 - g F_1(x)\} I \tag{2-12}$$

Finally, the correcting costs are incurred in the event that the cost report  $X > x$  and the process is out of

control. In the steady state, the probability of such an event is simply the second term in Equation (2-11), so that the expected cost per period of correcting the process equals:

$$C_K(x) = \frac{(1-g)[1 - F_2(x)]K}{1 - gF_2(x)} = \pi_1(x)(1-g)K \quad (2-13)$$

Thus the total expected cost per period of operating the controlled process is the sum of (2-10), (2-12) and (2-13), which can be expressed as follows:

$$\begin{aligned} C(x) &= C_O(x) + C_I(x) + C_K(x) \\ &= \mu_2 + \pi_1(x)\{(1-g)(K+I-g\Delta\mu) - gI\{F_1(x)\}\} \\ &= \mu_2 + \pi_1(x)\{a - bF_1(x)\}, \end{aligned} \quad (2-14)$$

where

$$a = (1-g)K + I - g\Delta\mu \stackrel{>}{<} 0, \quad (2-15)$$

$$b = gI \quad (I = 0 \text{ is uninteresting}).$$

#### 4. Optimal Policy

A natural question at this point is: "what control limit  $x^*$  is optimal?" Of course, the  $x$  that minimizes the total expected operating cost ( $C(x)$ ) is the optimal control limit. To get the optimal value of  $x$ , the first order derivative can be derived from Equation (2-14):

$$\frac{dC(x)}{dx} = \frac{d\pi_1(x)}{dx}\{a - bF_1(x)\} - b\pi_1(x)f_1(x) \quad (2-16)$$

$$\frac{d\pi_1(x)}{dx} = \frac{-(1-g)f_2(x)}{(1-gF_2(x))^2} \quad (2-17)$$

The cost  $C(x^*)$  is an extremum  $x^*$  such that  $dC(x^*)/dx = 0$ . That is:

$$(1-g)f_2(x^*)\{a-bF_1(x^*)\}+bf_1(x^*)\{1-F_2(x^*)\}1-gF_2(x^*)=0 \quad (2-18)$$

There are two possibilities; either  $x^*$  is such that  $\{a-bF_1(x^*)\} = 0$  and  $\{1-F_2(x^*)\} = 0$ ; in this case  $\pi_1(x^*) = 0$ , that is, the process runs uncontrolled forever. Or, since all other terms in Equation (2-18) are positive,  $x^*$  must lie in the open interval on which;

$$\{a - bF_1(x^*)\} < 0, \text{ that is, } F_1(x^*) > \frac{a}{b} \quad (2-19)$$

Intuitively we can understand that the optimal control limit should be a certain point where  $x^* > u_1$ . And for the  $x$  such that  $x < x^*$ , then  $C(x) > C(x^*)$ , also for the  $x$  such that  $x > x^*$ , then  $C(x) > C(x^*)$ . Therefore we can get optimal control limit  $x^*$  by trial and error.

#### B. BAYESIAN CONTROL METHOD [3]

In the Markovian control, the past cost reports don't have any effect on the current period's cost investigation decision. But in the Bayesian control it affects the current period's decision.

##### 1. Characteristic

This method was first suggested by Bierman, Fouraker and Jaedike [1]. The methodology was first developed by

Duvall [6] and later was expanded by Dykman [3]. This method is based on the Bayesian decision theory. The subjective prior probability can be converted to the posterior probability by additional information of periodic cost report, according to the Bayes' Theorem.

A two state, two action problem is assumed.

$\theta_1$ : in-control  
 $\theta_2$ : out-of-control  
 $a_1$ : investigate  
 $a_2$ : do not investigate

The decision on whether or not to investigate is based on reports of incurred costs. It is assumed that incurred costs are reported on a periodic basis. Thus, a do-not-investigate action implies that the activity is continued at least until the next cost observation is available. An additional assumption at this point is that a full investigation will always reveal the cause of an out-of-control action, which can then be immediately corrected.

The cost of investigation is assumed to be some constant  $I$ , and the present value of the savings obtainable from an investigation when the activity is out of control is  $L$  ( $L$  = Actual savings from the process—correction cost  $K$ ) where  $L > I$ ; otherwise an investigation would never be warranted. Values of  $L$  must be estimated before this method can be made operational.

Once  $L$  and  $I$  have been estimated, the payoffs in costs for the investigation problem in two-state form are as given

in Table 2-1. Typically these values will be of a magnitude such that the decision maker is willing to act on the basis of the expected values.

Table 2-1  
General Payoff Matrix

		state	
		$\theta_1$ : in-control	$\theta_2$ : out-of-control
$a_1$ : investigate	I	I-L	
$a_2$ : don't investigate	0	0	

To start, the process is assumed to be in a state of control but may move out-of-control with probability  $(1-q)$ . The process cannot shift from the out-of-control state to the in-control state. As in the Markovian control method, this situation can be represented by a Markov process with transition matrix:

$$\begin{matrix}
 & & 1 & 2 \\
 & & \begin{matrix} 1 & 2 \end{matrix} & \\
 1 & \begin{matrix} q & 1-q \end{matrix} & & \\
 2 & \begin{matrix} 0 & 1 \end{matrix} & & 
 \end{matrix} \tag{2-20}$$

2. Control Policy: Single Critical Probability for the Posterior Probability of the In-Control State

Assume now that the decision maker can and does have a (prior) subjective probability mass

function, call it  $f_0$ , over the states. Let the probability of state  $\theta_j$  in any period  $n$  ( $n = 1, 2, \dots$ ) be denoted by  $f_n(\theta_j)$ . Thus the initial probabilities are given by  $f(\theta_j)$ .

Under these assumptions, the "investigate" action should be taken at some stage  $n$  if its expected cost is less than the expected cost of delaying investigation. That is:

$$E[\hat{C}(a_1, \hat{\theta})] < E[\hat{C}(a_2, \hat{\theta})] = 0$$

where  $\hat{C}$  is the payoff random variable and  $\hat{\theta}$  is the state random variable. Substituting this means that

$$C[f_n(\theta_1)] + [I-L][1-f_n(\theta_1)] < 0 \quad (2-21)$$

An investigation is immediately called for if  $C[f_n(\theta_1)] + [I-L][1-f_n(\theta_1)]$  is negative. In each future period  $n$  ( $n = 1, 2, \dots$ ) a cost level  $\hat{x}$  is observed. Suppose that this cost level is more likely to occur when the process is out of control (see Figure 2-2). This increases the probability of state  $\theta_2$  at the expense of state  $\theta_1$ . If the probability attaching to the  $\theta_2$  is increased enough, then the expected loss from investigating will be less than the expected loss from not investigating and action  $a_1$  is preferred.

By the characteristics of the transition from in-control to out-of-control, the state probabilities should be adjusted. It can first be adjusted to reflect the effect of the transitional probabilities (2-20).

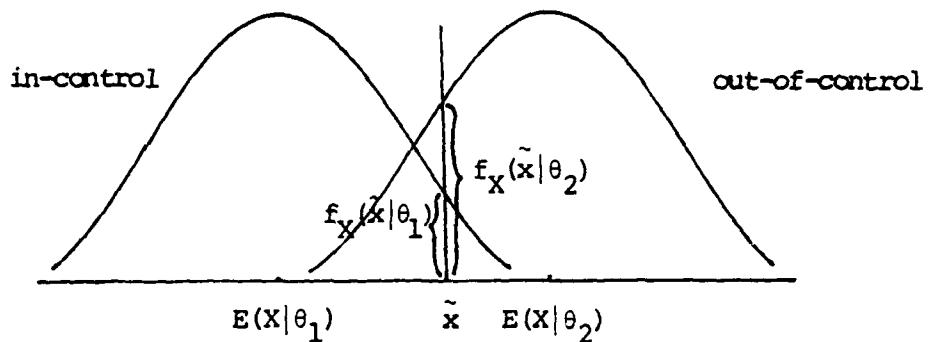


Figure 2-2: Cost Distributions

Assuming that if transition occurs before a cost observation, the results are:

$$f'_n(\theta_1) = gf_{n-1}(\theta_1) \quad (2-22)$$

$$f'_n(\theta_2) = (1-g)f_{n-1}(\theta_1) + f_{n-1}(\theta_2)$$

These are illustrated in Figure 2-3.

Then using these adjusted probabilities as prior probabilities of the period n, the revised probabilities given a cost report  $\tilde{x}$  can be obtained by Bayes' Theorem.

$$f_n(\theta_j) = f_n(\theta_j | \tilde{x}) = \frac{f_X(\tilde{x} | \theta_j) f'_n(\theta_j)}{\sum_{j=1}^2 f_X(\tilde{x} | \theta_j) f'_n(\theta_j)} \quad (2-23)$$

Note that state  $\theta_1$  cannot occur in period n if it did not exist in period n-1 and out of control state is assumed to be discovered if an investigation is made.

state in period n-1 and state probability	probability of state change	state in period n and state probability	revised state probability
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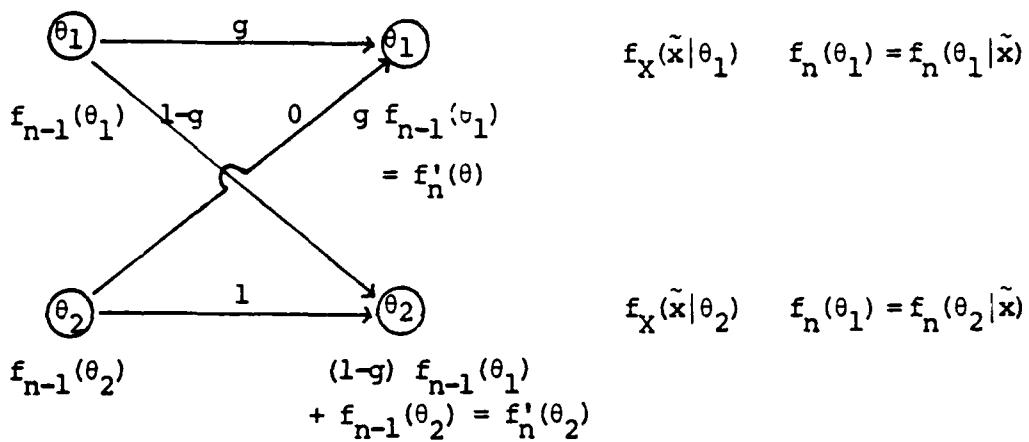


Figure 2-3: Illustration of Bayes' Theorem for Revised Probability

In order to avoid the expected value calculations each time a cost value is observed, the breakeven probability that equates the two actions can be obtained. To do so, let the revised state probability for state  $\theta_1$  after n cost observations be given by  $f_n(\theta_1)$ . Thus  $f_n(\theta_2) = 1 - f_n(\theta_1)$ . Then solving for the expected costs of investigation gives:

$$E[\hat{C}(a, \hat{\theta})] = Cf_n(\theta_1) + [I-L][1 - f_n(\theta_1)] \quad (2-24)$$

If this expectation is less than zero, the process is a candidate for investigation; while it exceeds zero, the

process is not. Setting the expectation given by Equation (2-24) equal to zero and solving the equation gives the breakeven value, or the indifferent point:

$$f_n(\theta_1) = \frac{L - I}{L} \quad (2-25)$$

For example, we assume that  $L$  equals 12,000 and  $I$  equals 2,000, then the breakeven probability is 0.83  $((12000-2000)/12000)$ . This breakeven probability is denoted by  $q$  in this paper. If  $f_n(\theta_1) < q$ , an investigation is signaled, otherwise it is not. Note that the breakeven value is independent of stage  $n$  and therefore relevant to all time periods.

#### C. DISTINCTION BETWEEN TWO METHODS

The major distinction between the two methods are the criteria of whether or not to investigate the process. The Markovian Control method relies on the critical limit ( $x^*$ ) to decide whether or not to investigate the process. The process is investigated simply because the reported cost exceeds a critical value  $x^*$ ; the history of cost reports is forgotten. The critical limit can be obtained by trial and error. On the other hand, the Bayesian Control method relies on the critical probability ( $q$ ) to decide whether or not to investigate the process. In this method, the manager keeps track of the probability of the process being in-control at the time of the next cost report; the periodic cost reports serve to update this probability and the process is investigated

whenever the updated probability is less than the critical probability.

Although each method has a different criterion for the investigation of the process, both methods aimed to minimize long run expected cost. Thus we have to resort to the long run expected cost in comparing the effectiveness of the two methods. An analytical approach to the comparison of effectiveness of the two methods was presented in the article "Cost Variance Investigation: Markovian Control versus Optimal Control" [7], by Dittman and Prakash in 1979. In this article they concluded that "it is observed that Markovian Control (Dittman-Prakash policy) performs almost as well as the Optimal Control (the best Bayesian policy) unless the in-control cost has substantially greater dispersion than the out of control" [7: p. 358]. The best Bayesian policy means that the process is controlled by optimal critical probability ( $q^*$ ), which can be obtained from solving dynamic programming suggested by Kaplan, rather than controlled by the breakeven probability ( $q$ ) (which is simpler than optimal critical probability). Dittman and Prakash assumed that the Optimal Control was the best control method under the criterion of minimizing long run expected cost. But Markovian Control is less complex than Optimal Control to apply. Under these assumptions they measured the opportunity cost of simplicity in the Markovian Control.

This analytical approach did not do anything with the actual periodic cost report. How the actual cost reports

behave against the theoretical control criteria can be examined only by an empirical study (simulation). Cost report in a period is a random sample from the defined probability distribution and the manager should decide whether or not to investigate the process by this random sample.

In this paper Bayesian Control with the breakeven probability is compared with the Markovian Control. The two methods are selected for comparison for the following two reasons:

- (1) both are easy to apply in a realistic setting and therefore the opportunity cost of simplicity can be glossed over;
- (2) no research has been done to compare the relative effectiveness of the Markovian Control and the breakeven critical probability Bayesian method.

### III. DESCRIPTION OF THE SIMULATION MODEL

#### A. DECISION-MAKING ENVIRONMENT

The preceding chapter described the Bayesian and the Markovian decision models. However the relative effectiveness of the two models remains to be tested. Different decision-making environments, such as cost savings from an investigation, different distributions of cost performance under the in-control or out-of-control state, etc., may have different effect on the usefulness of the two decision models.

In the Markovian Control model the process cost was expressed by Equation (2-14):

$$C(x) = \mu_2 + \pi_1(x)\{a - bF_1(x)\},$$

where

$$a = (1-g)K + I - g\Delta\mu,$$

and

$$b = gI.$$

If the process is never investigated, the long run expected process cost per period is  $\mu_2$  (the mean cost of out-of-control state). Therefore in the above equation, the term  $\pi_1(x)\{a - bF_1(x)\} = s(x)$  should be negative for control to be worthwhile. Since  $F_1(x)$  can never be greater than one and  $\pi_1(x)$  can never be negative, if the value  $a/b$  is such that

$a/b \geq 1.0$ , the term  $s(x)$  is necessarily positive. It means that the optimal policy for this process is "never investigate". On the other hand, if the value of  $a/b$  is much smaller than -1.0, the optimal policy is "always investigate". Thus, the relevant range for the value of  $a/b$  should be less than one but not too much less than -1.0.

In the Bayesian Control model, the control point (or breakeven probability) was expressed by Equation (2-23)  $(\frac{L-I}{L})$ . In the same context, the extreme values of breakeven probability (close to zero or close to one) are not proper for comparison with the Markovian Control model.

#### B. PROBABILITY DISTRIBUTIONS OF IN-CONTROL AND OUT-OF-CONTROL STATES

In order to evaluate the relative effectiveness of the two models under study, we can consider various combinations of probability distributions of in-control and out-of-control states. For example, dispersion of in-control state can be larger or smaller than or the same as that of the out-of-control state.

The probability distributions assumed in this simulation study are as follows.

(a) Equally dispersed in-control and out-of-control states with low degree of overlap--in this case, the mean cost of an in-control state is assumed to be \$100 with a standard deviation of \$5, and the mean cost of an out-of-control state is assumed to be \$120 with a standard deviation of \$5.

Figure 3-1 depicts this situation.

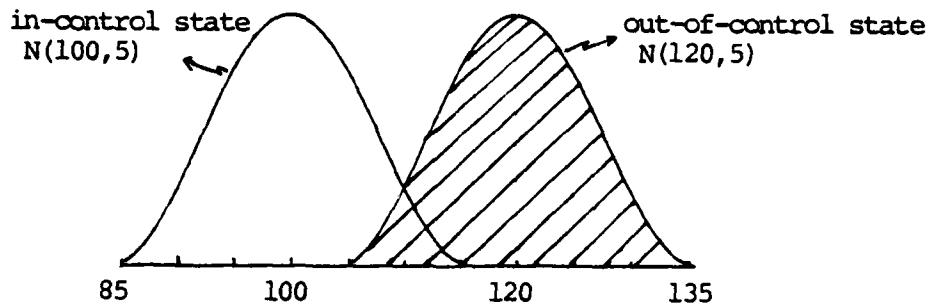


Figure 3-1: Equally Dispersed Low Overlap

(b). In-control state less dispersed than out-of-control state--in this case, the mean cost of an in-control state is assumed to be \$100 with a standard deviation of \$5 and the mean cost of an out-of-control state is assumed to be \$120 with a standard deviation of \$30.

Figure 3-2 depicts this situation.

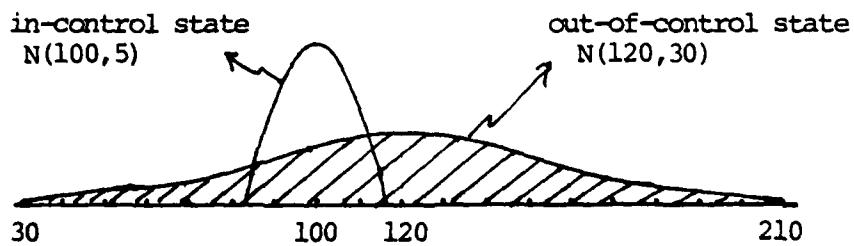


Figure 3-2: In-Control Less Dispersed Than Out-of-Control

(c). In-control state more dispersed than out-of-control state--in this case, the mean cost of an in-control state is assumed to be \$100 with a standard deviation of \$30 and the

mean cost of an out-of-control state is assumed to be \$120 with a standard deviation of \$5.

Figure 3-3 depicts this situation.

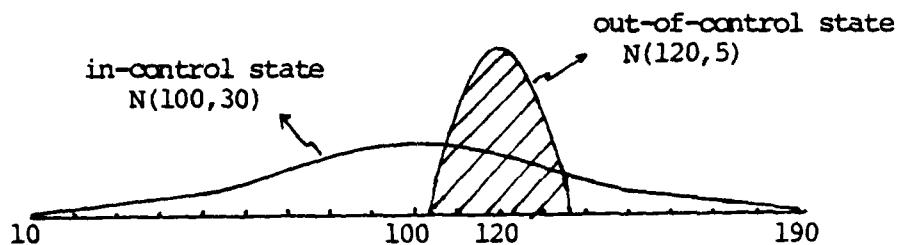


Figure 3-3: In-Control More Dispersed Than Out-of-Control

(d). Equally dispersed in-control and out-of-control states with large degree of overlap--in this case, the mean cost of an in-control state is assumed to be \$100 with a standard deviation of \$30 and the mean cost of an out-of-control state is assumed to be \$120 with a standard deviation of \$30.

Figure 3-4 depicts this situation.

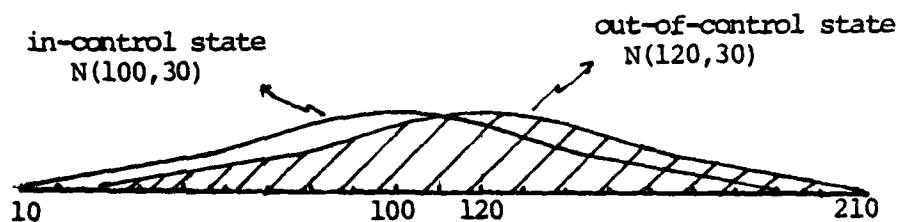


Figure 3-4: Equally Dispersed Large Overlap

For the following two cases, the probability distribution of the out-of-control state is assumed to have a gamma distribution. The gamma distribution is not symmetric, it is skewed to the left from the mean (Figure 3-5). The gamma distribution is decided by two parameters  $(\gamma, \lambda)$ . Since the parameter  $\gamma$  affects the shape of the distribution, we can make various shapes by selecting proper  $\gamma$  according to the characteristic of the cost process under consideration. For some situations, the use of gamma distributions could be more practical and more similar to the actual process.

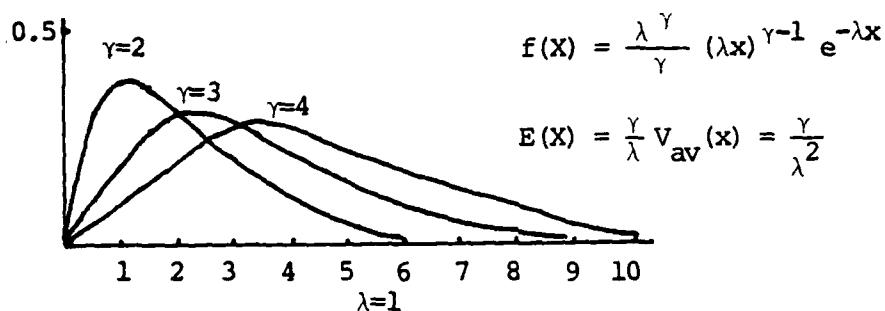


Figure 3-5: Gamma  $(\gamma, 1)$  Distributions

(e). Normal distribution with small dispersion in in-control state and gamma distribution in out-of-control state--in this case the in-control state is assumed to be a normal distribution with a mean of \$120 and  $\lambda$  equal to one. Since parameter  $\gamma$  affects the shape of the gamma distribution, the shape is approximately symmetric when  $\gamma$  exceeds 50. To simulate a process skewed to the left, two is assumed for the parameter  $\gamma$  and then it is transformed to the mean of 120. The formula chosen for the transformation is  $Y = 85 + 17.5X$ ; where  $X$  is a

Gamma (2,1). Then the mean of  $Y$  is 120 and the variance of  $Y$  is 612.5 (standard deviation is 24.75). We can make different shapes of Gamma (2,1) with various mean and variance by adding and multiplying Gamma (2,1). For convenience,  $G(120,1)$  refers to the transformed distribution in this paper.

Figure 3-6 depicts this situation.

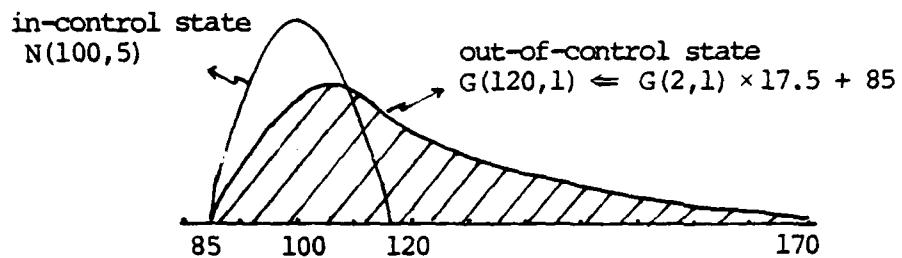


Figure 3-6: Narrow Normal In-Control and Gamma Out-of-Control

(f). Normal distribution with large dispersion in the in-control state and gamma distribution in the out-of-control state--in this case the in-control state is assumed to be a normal distribution with a mean of \$100 and a standard deviation of \$30 and the out-of-control state is assumed to be the same as case 5.

Figure 3-7 depicts this situation.

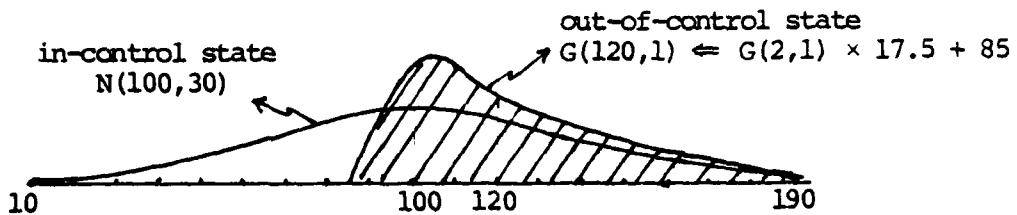


Figure 3-7: Wide Normal In-Control and Gamma Out-of-Control

### C. BOUNDARY CONDITIONS BETWEEN OUT-OF-CONTROL STATE AND IN-CONTROL STATE

We can further consider different boundary conditions in which the status of a cost process at the beginning of each period is assumed. Both the Bayesian Control model and the Markovian Control model assume that an investigation always uncovers the reason why the process has deteriorated to the out-of-control state and that the causal factor is corrected. However, we can suppose a boundary condition between out-of-control and in-control state. If we investigate the process when the process is in control, we may or may not know to what degree the process has transferred into the out-of-control state. If an investigation simply reveals whether the process is in-control or out-of-control but does not determine to what extent the process is drifting toward out-of-control state, the probability of the in-control state doesn't return to one at the beginning of the next period by an investigation in the preceeding period. In other words, the process will operate one more period in the same condition as at the end of the preceeding period, i.e., it is drifting into out-of-control state to some extent. Thus the probability of the in-control state at the beginning of the next period may not return to one; say condition (2).

On the other hand, we can also suppose that the above boundary condition cannot exist and that an investigation always reveals the status of the cost process and the probability of the in-control state is returned to one after an investigation; say condition (1).

#### D. SAVINGS FROM AN INVESTIGATION

Since  $L$ , the loss for failing to conduct an investigation or savings from a productive investigation, is specifically considered in the Bayesian Control model but not in the Markovian Control model, this study also examines the effect of varying the value of  $L$  on the effectiveness of each model.

The three values selected for this study (25, 45, 100) are admittedly arbitrary, but the purpose is to identify the direction of the change in effectiveness, not the "correct" and "efficient" absolute number of each model, which really does not exist except by subjective estimation. When consistent patterns in model effectiveness are discovered, additional values of  $L$  are simulated to ascertain the pattern.

#### E. SUMMARY OF CONDITIONS SIMULATED

Under each of the two boundary conditions there are 18 different combinations of cases (data points) according to the assumptions made on different cost distributions and savings. The 18 data points are summarized in Table 3-1.

The values for the investigation cost ( $I$ ), correction cost ( $K$ ), transition probability ( $g$ ) and mean differential cost ( $\Delta\mu$ ) are assumed to be 15, 5, 0.9 and 20 respectively in all cases. Calculated value of  $a$  and  $b$  are -2.5 and 13.5, respectively. Therefore the value of  $a/b$  (-0.185) is not an extreme case.

#### F. SIMULATION PROCEDURE

The simulation procedure is summarized in Appendix A in the form of computer program flow chart. The computer program

Table 3-1  
18 Different Data Points Considered

$I = 15, K = 5, g = 0.9$

in-control sav- ings		N(100,5)			N(100,30)		
		L = 25	L = 45	L = 100	L = 25	L = 45	L = 100
N(120,5)	N(100,5)	N(100,5)	N(100,5)	N(100,30)	N(100,30)	N(100,30)	N(100,30)
	N(120,5)						
	L = 25	L = 45	L = 100	L = 25	L = 45	L = 100	L = 100
N(120,30)	N(100,5)	N(100,5)	N(100,5)	N(100,30)	N(100,30)	N(100,30)	N(100,30)
	N(120,30)						
	L = 25	L = 45	L = 100	L = 25	L = 45	L = 100	L = 100
G(120,1)	N(100,5)	N(100,5)	N(100,5)	N(100,30)	N(100,30)	N(100,30)	N(100,30)
	G(120,1)						
	L = 25	L = 45	L = 100	L = 25	L = 45	L = 100	L = 100

N: Normal distribution      G: Gamma distribution

G(120,1): Refers to  $G(2,1) \times 17.5 + 85$

is included in Appendix B. One hundred periods are simulated in each sample and 50 samples are used to estimate the mean differential cost ( $\bar{\bar{X}}_D$ ) between the two decision models. Alternatively, one can interpret that the simulation result is based on 5000 iterations, which is more than enough to guarantee stability of the simulation result and provide the basis for the evaluation of the long-term effectiveness of the decision models under study.

#### IV. DISCUSSION OF RESULTS

Results of the implementation of the simulation model for the data points and conditions, that were explained in Chapter III, are summarized in Table 4-1.

In Table 4-1, the positive value of  $\bar{X}_D$  means that the incurred cost for Bayesian Control is less than that of Markovian Control and the negative means the converse. To test whether or not the values are statistically significant, Table 4-2 shows the 95% confidence interval for each value of  $\bar{X}_D$ . We can say that the value of  $\bar{X}_D$  is statistically significant if the confidence interval doesn't include zero. Otherwise, we can't say that. Furthermore, we have to compare the value of  $\bar{X}_D$  with the mean difference cost between in-control and out-of-control state (\$20 in this study). This comparison enables us to see the relative magnitude of the value of  $\bar{X}_D$ .

We now can look for significant patterns in the tabulated results. A relative magnitude of differential cost between the two control methods can be used to figure out whether or not the differential cost is practically significant. The differential costs of the two control methods are affected by the mean cost difference of the two states ( $\Delta\mu$ ) to some extent, but not affected by the absolute amount of a cost report.

Table 4-1  
Simulation Results

(a) N(100,5) N(120,5)										
Data Points	Savings (L)	M <sub>C</sub>	P <sub>C</sub>	$\bar{\bar{X}}_D$	Condition (1)			Condition (2)		
					R.M. (%)	S <sub>X̄_D</sub>	$\bar{\bar{X}}_D$	R.M. (%)	S <sub>X̄_D</sub>	
1	25	112.50	0.40	0.176	0.9	0.262	0.175	0.9	0.260	
2	45	112.50	0.67	0.210	1.1	0.256	0.209	1.0	0.257	
3	100	112.50	0.85	0.185	0.9	0.284	0.178	0.9	0.284	
(b) N(100,5) N(120,30)										
4	25	112.65	0.40	0.996	5.0	0.708	0.831	4.2	0.762	
5	45	112.65	0.67	-0.249*	1.2	0.941	-1.028	5.1	1.180	
6	100	112.65	0.85	-3.915	19.6	1.020	-4.911	24.6	1.169	
(c) N(100,30) N(120,5)										
7	25	117.25	0.40	1.902	9.5	1.204	2.029	10.1	1.342	
8	45	117.25	0.67	2.037	10.2	1.141	2.168	10.8	1.269	
9	100	117.25	0.85	2.481	12.4	1.023	2.613	13.1	1.151	
(d) N(100,30) N(120,30)										
10	25	130.95	0.40	0.580	2.9	1.043	1.300	6.5	1.123	
11	45	130.95	0.67	1.590	8.0	1.110	1.964	9.8	1.136	
12	100	130.95	0.85	0.283*	1.4	1.094	-0.023*	0.1	1.246	
(e) N(100,5) G(120,1)										
13	25	111.35	0.40	0.366	1.8	0.734	0.434	2.2	0.842	
14	45	111.35	0.67	-0.645	3.2	0.822	-1.303	6.5	0.972	
15	100	111.35	0.85	-4.266	21.3	0.938	-4.754	23.8	1.230	
(f) N(100,30) G(120,1)										
16	25	124.50	0.40	1.297	6.5	1.199	2.313	11.6	1.297	
17	45	124.50	0.67	2.504	12.5	0.980	3.426	17.1	1.113	
18	100	124.50	0.85	2.315	11.6	1.126	2.791	14.0	1.305	

Table 4-1 (Cont'd)

$\bar{\bar{X}}_D$ : Estimate of  $\bar{X}_D$ ; Arithmetic mean of incurred cost of Markovian Control minus incurred cost of Bayesian Control, derived from fifty mean ( $\bar{X}_D$ ) of one hundred iterations.

R.M. (%): Relative magnitude of  $\bar{\bar{X}}_D$  to the mean difference cost between in-control and out-of-control state.

$$(\frac{\bar{\bar{X}}_D}{\mu_2 - \mu_1} \times 100) = \frac{\bar{X}_D}{20} \times 100$$

$S_{\bar{\bar{X}}_D}$ : Estimated standard deviation of  $\bar{\bar{X}}_D$  ( $S_{\bar{\bar{X}}_D} = \frac{S_{\bar{X}_D}}{\sqrt{49}}$ )

Table 4-2  
95% Confidence Interval of  $\bar{x}_D$

Data Points	Conditions (1)	Condition (2)
1	0.101 - 0.251	0.101 - 0.249
2	0.137 - 0.283	0.136 - 0.282
3	0.104 - 0.266	0.097 - 0.259
4	0.795 - 1.197	0.614 - 1.048
5	-0.528 - 0.031*	-1.363 - -0.692
6	-4.218 - -3.611	-5.243 - -4.578
7	1.544 - 2.260	1.647 - 2.411
8	1.698 - 2.376	1.807 - 2.529
9	2.177 - 2.785	2.285 - 2.941
10	0.283 - 0.877	0.980 - 1.620
11	1.274 - 1.906	1.641 - 2.287
12	-0.028 - 0.594*	-0.377 - 0.332*
13	0.157 - 0.575	0.194 - 0.674
14	-0.879 - -0.411	-1.580 - -1.026
15	-4.533 - -3.999	-5.104 - -4.404
16	0.956 - 1.638	1.950 - 2.676
17	2.225 - 2.783	3.109 - 3.743
18	1.995 - 2.635	2.420 - 3.108

t value (95%, 49 degrees of freedom): 2.012

\* data cell that is statistically indifferent

A. **EQUALLY DISPERSED IN-CONTROL AND OUT-OF-CONTROL STATE WITH LOW DEGREE OF OVERLAP (N(100,5),N(120,5))**

This case included data points 1,2,3. Bayesian Control performed better than Markovian Control in all the data points of this case. But the relative magnitudes of differential cost of the two methods were small. The greatest magnitude in Table 4-1(a) was 1.1 percent of the mean difference cost between the two states. Therefore, it is hard to conclude that the one method performs better than another.

Dispersion of the distribution of states can be interpreted as a degree of uncertainty in a probability distribution. The degree of uncertainty in this case is relatively small for both states. It means that the chance of a manager committed to type I, or type II error is very low. Therefore small magnitude of differential cost of the two methods can be attributed to that reason.

B. **IN-CONTROL STATE LESS DISPERSED THAN OUT-OF-CONTROL STATE (N(100,5),N(120,30))**

This case included data points 4,5,6. This case is more likely to occur in the real world. It is reasonable that the outcome is an in-control state is more certain than that of an out-of-control state.

In this case, Markovian Control performed better than Bayesian Control for larger values of L, but the reverse is true for lower values of L. In the medium range, however, the result is inconclusive, as indicated by the \*. The relative performance of the Markovian Control compared to the Bayesian Control was bettered as the amount of savings

increases. To see the trend of the differential cost between the two methods, differential costs for three additional values of savings were obtained. Figure 4-1 shows this result.

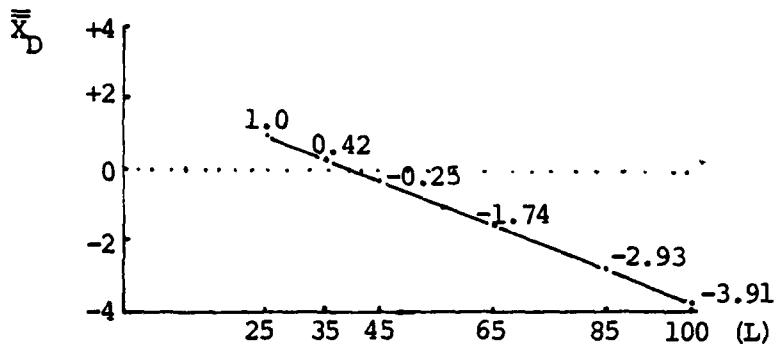


Figure 4-1: Relationship Between  $\bar{x}_D$  and L(B)

The relative magnitude of Markovian Control's better performance was relatively large in the larger values of L; it was 19.6% under condition (1) and 24.6% under condition (2). On the other hand, the relative magnitude of Bayesian Control's better performance in the lower values of L was around 5%.

Therefore, we can say that the Markovian Control performs better than Bayesian Control in this case, especially when the value of L is large.

#### C. IN-CONTROL STATE MORE DISPERSED THAN OUT-OF-CONTROL STATE (N(100, 30), N(120, 5))

This case included data points 7, 8, 9. This case is less likely to occur in the real world.

In this case, the Bayesian Control performed better than the Markovian Control in all the data points. And the relative

performance of the Bayesian Control compared to the Markovian Control was bettered as the amount of savings increases. This trend is the reverse of Case B discussed above. Figure 4-2 shows the relationship between  $\bar{X}_D$  and L.

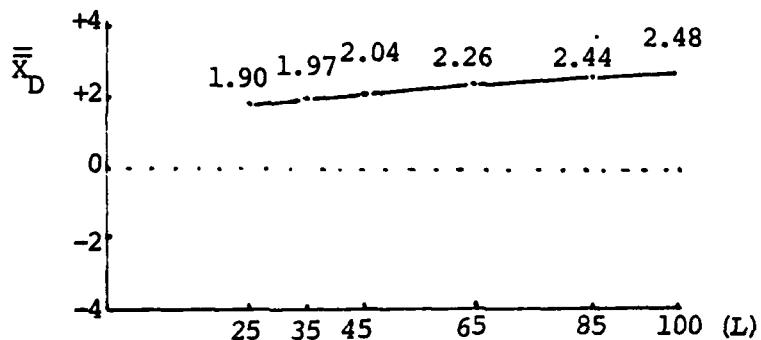


Figure 4-2: Relationship Between  $\bar{X}_D$  and L(C)

The relative magnitudes of Bayesian Control's better performance ranged from 9.5% to 13.1%.

Therefore, we can say that the Bayesian Control performs better than the Markovian Control in this case.

D. EQUALLY DISPERSED IN-CONTROL AND OUT-OF-CONTROL STATE WITH LARGE DEGREE OF OVERLAP (N(100,30),N(120,30))

This case included data points 10,11,12. In this case, the Bayesian Control performed better than the Markovian Control in the middle range of L values simulated. However the advantage of the Bayesian Control model appears to decrease as the assumed value of L gets higher or lower.

Figure 4-3 shows the relationship between  $\bar{X}_D$  and L.

The relative magnitude of Bayesian Control's better performance was greatest in the medium values of L; it was 8%

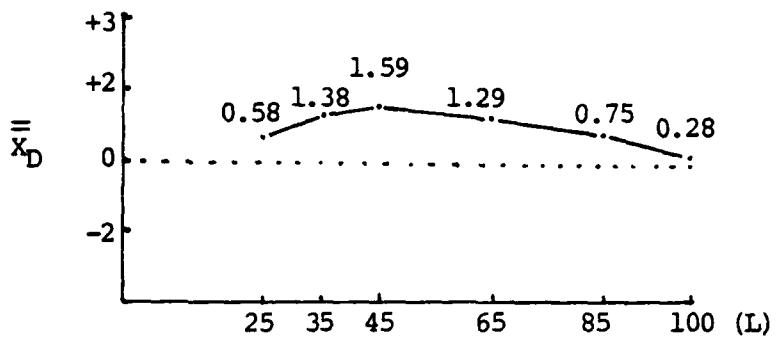


Figure 4-3: Relationship Between  $\bar{\bar{X}}_D$  and  $L(D)$

and 9.8% respectively under the two boundary conditions assumed. Therefore, the results of this case is somewhat inconclusive.

E. NORMAL DISTRIBUTION WITH SMALL DISPERSION IN-CONTROL STATE AND GAMMA DISTRIBUTION IN OUT-OF-CONTROL STATE ( $N(100,5), G(120,1)$ )

This case included data points 13,14,15. In this case, the Bayesian Control performed better than the Markovian Control in the lower values of  $L$  and the Markovian Control performed better than the Bayesian Control in the medium and larger values of  $L$ . Figure 4-4 shows the relationship between  $\bar{\bar{X}}_D$  and  $L$ .

The relative performance of the Markovian Control compared to the Bayesian Control was bettered as the values of  $L$  increases.

The relative magnitudes of Bayesian Control's better performance were 2% in the lower values of  $L$  and those of Markovian Control's better performance were around 5% in the medium values of  $L$  and around 22% in the larger values of  $L$ .

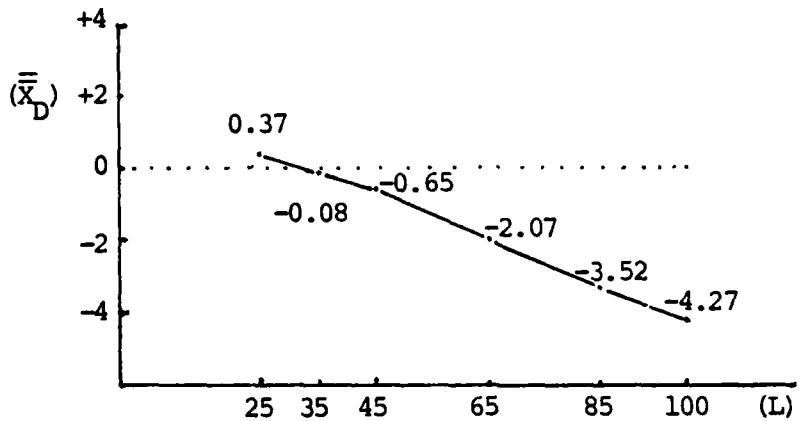


Figure 4-4: Relationship Between  $\bar{\bar{X}}_D$  and L (E)

Therefore we can say that the Markovian Control generally performed better than the Bayesian Control in this case.

F. NORMAL DISTRIBUTION WITH LARGE DISPERSION IN-CONTROL STATE AND GAMMA DISTRIBUTION IN OUT-OF-CONTROL STATE (N(100, 30), G(120, 1))

This case included data points 16, 17, 18. In this case, the Bayesian Control performed better than the Markovian control in all the data points. The relative performance of the Bayesian Control compared to the Markovian Control was best in the medium values of L. Figure 4-5 shows the relationship between  $\bar{\bar{X}}_D$  and L.

The relative magnitudes of Bayesian Control's better performance were around 9% in the lower values of L, around 15% in the medium values of L and around 12% in the larger values of L.

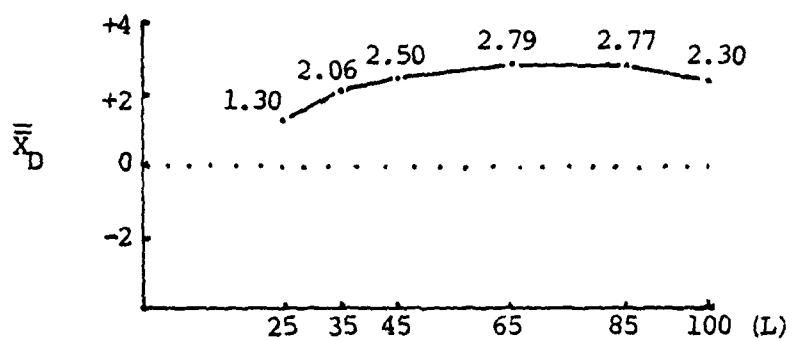


Figure 4-5: Relationship Between  $\bar{\bar{X}}_D$  and L(F)

We can say that the Bayesian Control performed better than the Markovian Control in this case.

#### G. COST REPORTS OF THE PROCESS

As a by-product of the simulation, a mean of cost reports derived from the 5000 iterations was calculated for each case. Cost reports depend on the combination of cost distribution of the two states. Table 4-3 shows those results.

Table 4-3  
Means of Cost Reports

Case	In- Control State	Out-of Control State	Markovian Control		Bayesian Control	
	Condition (1)	Condition (2)	Condition (1)	Condition (2)	Condition (1)	Condition (2)
a	N(100,5)	N(120,5)	105.15	105.18	102.07	102.15
b	N(100,5)	N(120,30)	105.93	105.97	103.64	105.38
c	N(100,30)	N(120,5)	104.22	105.98	102.33	102.34
d	N(100,30)	N(120,30)	105.31	107.37	104.07	105.04
e	N(100,5)	G(120,1)	106.09	106.20	103.98	105.93
f	N(100,30)	G(120,1)	105.85	107.72	103.41	103.65

In Table 4-3, we can see that means of cost reports are slightly different from case to case. The mean of cost reports were not affected much by the distributions of the two states in the controlled process.

Cost differences between the mean of Bayesian Control's cost reports and the mean of Markovian Control's cost reports do not affect the differential cost of the two methods ( $\bar{X}_D$ ) in the simulation model of this paper.

## V. CONCLUSION

Suggestions to use statistical models to investigate cost variances originated from the statistical quality control techniques in industry. However, the application of statistical cost variance investigation methods is minimal so far. Kochler reported that "in some general inquiry from some prominent corporations, I was unable to find single use of statistical procedures for variance control" [13]. He attributes this paucity of applications not to the inherent inapplicability of such procedures but to "the fact that accountants have not recognized a conceptual distinction between a significant and an insignificant variance" [13]. Therefore, he proceeds to advocate the use of simple testing procedures.

The optimal cost investigation policy developed under the Dynamic Programming approach (Kaplan) [6] obviously is difficult to apply. This study compares two statistical models that are relatively easy to apply. The purpose of this study is to examine the relative effectiveness of the Bayesian Control model and the Markovian Control model under different assumptions.

The results of the simulation study shed some light on the relative effectiveness of two well developed models under various conditions.

In Table 4-1 we can see that the values of  $\bar{\bar{X}}_D$  were not much different between condition (1) and condition (2) for

each data point. It means that whether or not the probability of the in-control state returns to one after an investigation doesn't have much influence on the performance of the two models. In other words, different assumptions about the probability of the in-control state after an investigation are not an important issue.

Based on the analysis of the simulation results, some general conclusions regarding the relative effectiveness of the two decision models can be made as follows:

(1) When the distributions of the two states (in-control and out-of-control) are identical (Cases A and D), both decision models are virtually equally effective, although the Bayesian model has a slight edge when the distributions are widely dispersed (Case D).

(2) When the distributions of the two states are different, the effectiveness of a cost variance investigation decision model seems to be dependent on the cost distribution of the in-control state.

a) If the cost distribution of the in-control state is normal with low degree of dispersion (Cases B and E), the Markovian Control model has a distinct advantage, especially when the value of  $L$  is high. This finding is generally consistent with Dittman and Prakash's conclusion [7].

b) If the cost distribution of the in-control state is normal but the dispersion is wider than that of out-of-control state (Cases C and F), the Bayesian Control model

seems to have an edge. One must admit, however, that Cases C and F are less likely to occur in the real world.

(3) The assumed value of L plays a significant role in the relative effectiveness of a decision model in all but Case A. The larger the value of L is assumed, the better the relative performance of the Markovian model in Cases B and E (normal distribution with narrow dispersion for in-control state). Curvilinear relationships are found in Cases C and D (normal distribution with wide dispersion for in-control state).

In summary we observed that the Markovian Control decision model seems to be as effective as, if not better than, the Bayesian Control model in those conditions more likely to be found in the real world. However the Bayesian Control model seems to be more effective in those special cases where the cost distribution of an in-control state is widely dispersed.

APPENDIX A  
PROCEDURES OF THE SIMULATION MODEL

The procedures of the simulation model can be explained by a step-by-step discussion of the flow chart of it (Figure A-1). For the convenience of explanation a alphabetical label, under which each procedure is explained, is attached to each procedure.

(a) For the simulation the variables that should be estimated outside of the model are given as external factors.

$L$ (long run expected savings from an investigation) is not an easy matter to determine. But it can be determined by the method that was suggested by Duvall [8].

$I$ (cost of investigation),  $g$ (the probability that in-control state in the current period still remains in an in-control state, in the next period), and  $K$ (correction cost when the process is out-of-control state) can be estimated from the past experience and historical data of the process.

Then the distributions of two states can also be determined by past experience.

For example, during the first iteration we assume that  $L$  equals 60,  $I$  equals 15,  $K$  equals 5,  $g$  equals 0.9, the distribution of the in-control state is  $N(100,5)$  and the distribution of the out-of-control state is  $N(120,5)$ . This set of numbers will be used in the discussion of steps that follow for illustration purposes.

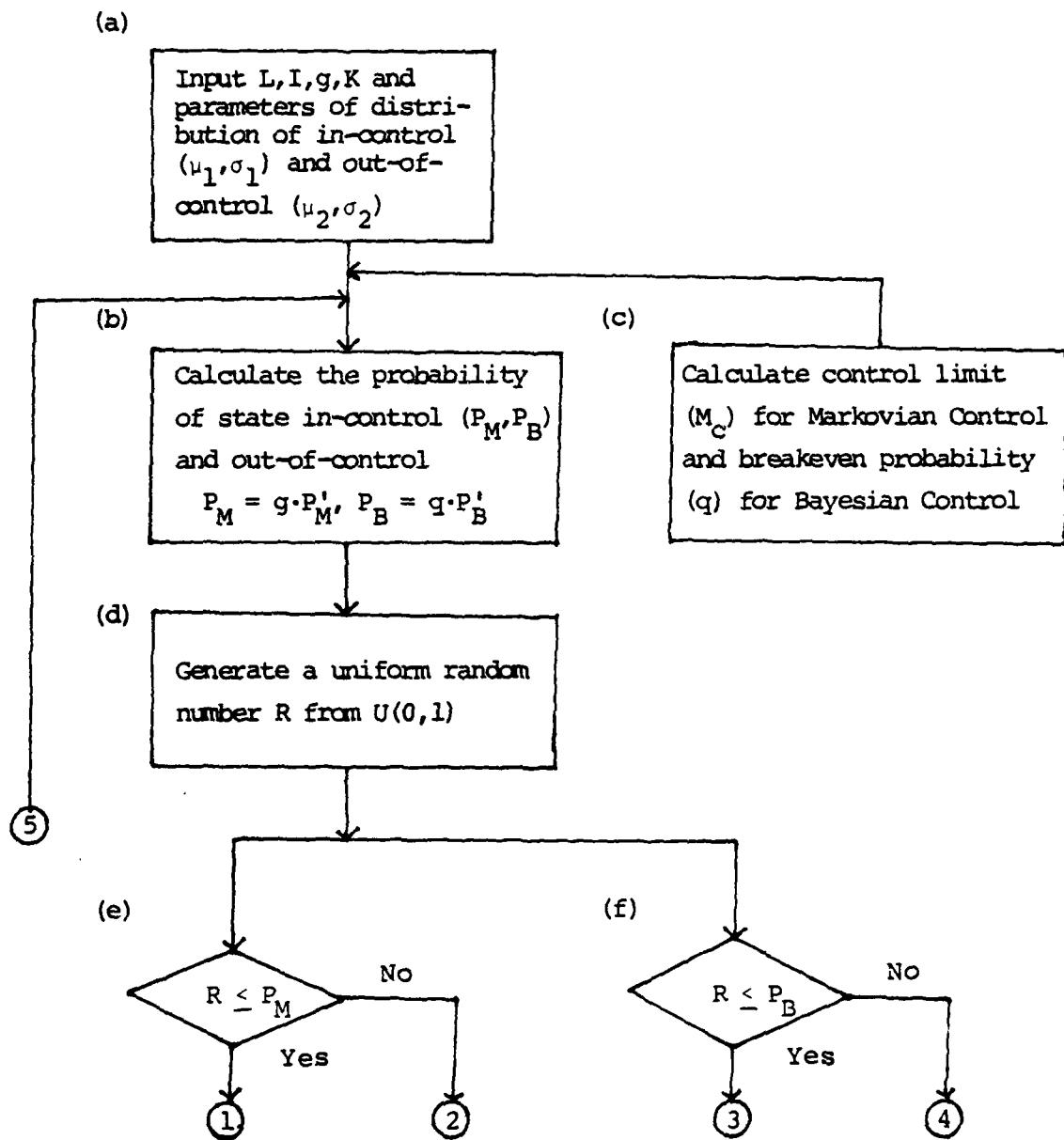


Figure A-1: Flow Chart

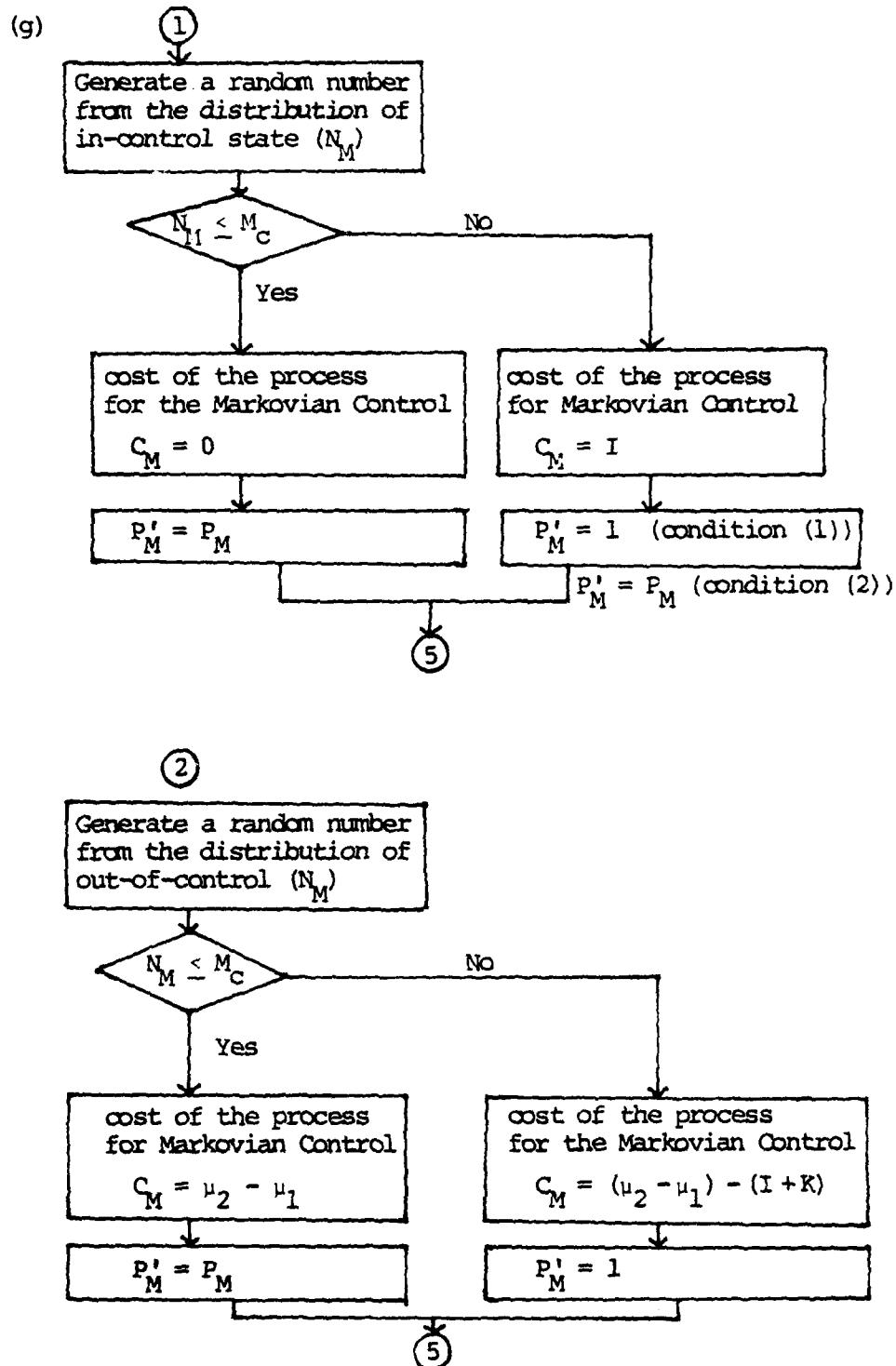


Figure A-1 (Cont'd)

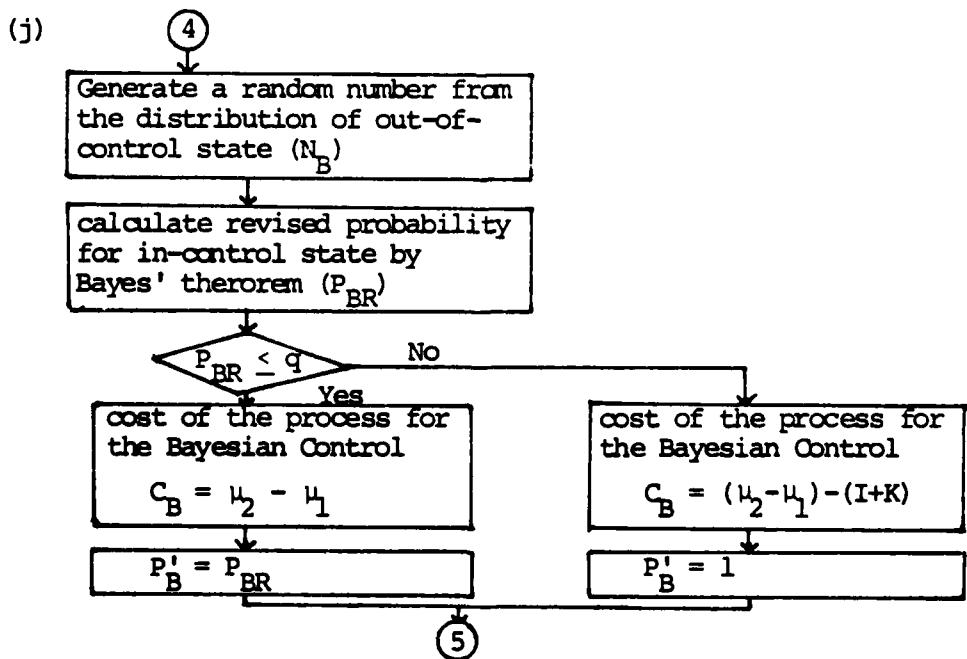
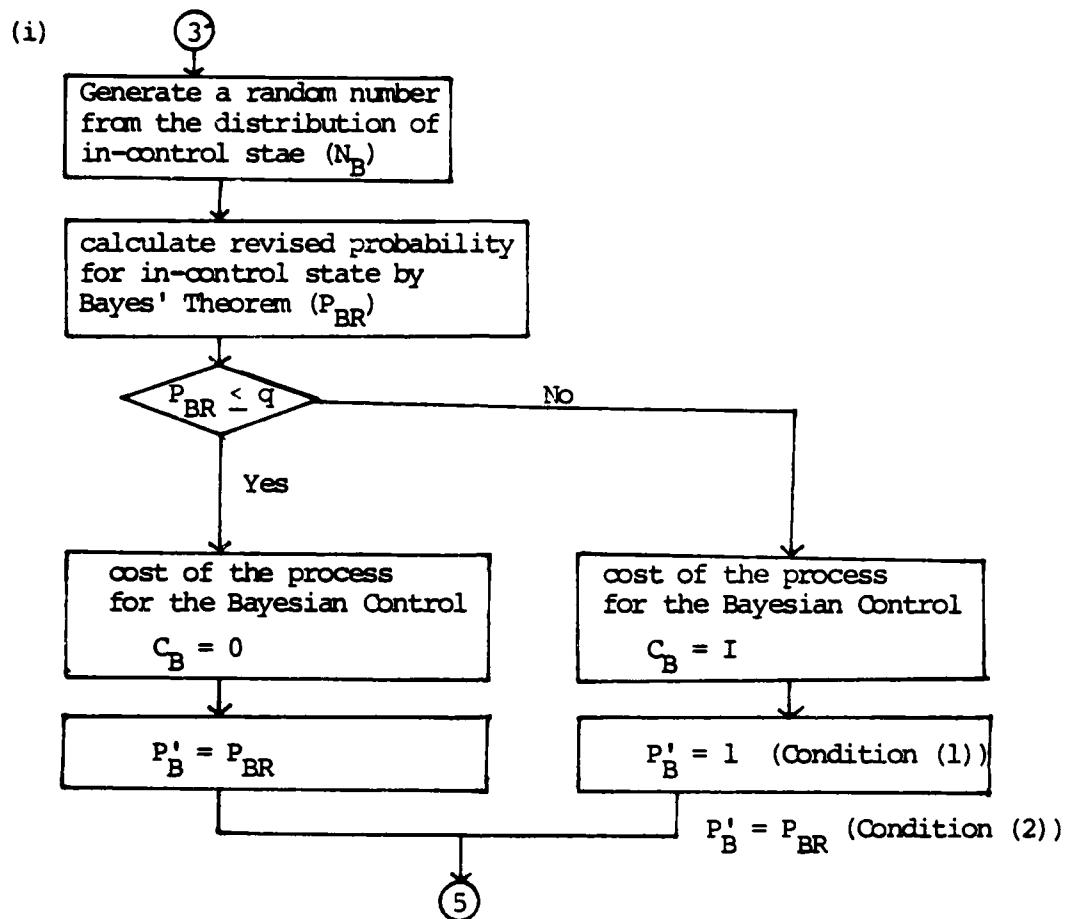


Figure A-1 (Cont'd)

(b) Then we can assume that the process starts with in-control state; probability of in-control state equals one and the probability of out-of-control state equals zero. A cost report comes from the process which was transformed by the Transition Matrix from the prior period's states. Therefore we can calculate the probability of the in-control state of the process from which the cost report comes out.

For example, for period one,

$$P_M = 1 \times 0.9 = 0.9;$$

$$P_B = 1 \times 0.9 = 0.9.$$

(c) In this step, we calculate the optimal control limit ( $M_c$ ) for the Markovian Control by trial and error method. Total cost of the process decreases as the value of the control limit increases until a certain point. Beyond this point, total cost of the process increases as the value of the control limit increases. Therefore we can find a point which minimizes total process cost. It is a search process in the computer program (see Appendix B computer program).

For example, the total cost of the process for a given control limit of Markovian Control is as follows: suppose that a given control limit is 110.

From Equation 2-15;

$$a = (1-g)K + I - g\Delta\mu = (0.1 \times 5) + 15 - (0.9 \times 20) = -2.5;$$

$$b = gI = 0.9 \times 15 = 13.5.$$

From Equation 2-8:

$$\pi_1(x) = \frac{1 - F_2(x)}{1 - qF_2(x)} = \frac{1 - F_2(110)}{1 - 0.9F_2(110)};$$

$$F_2(110) = \Pr(X \leq 110 | N(120, 5))$$

We can obtain the value of  $F(110)$  from the normal probability table. For computer simulation, we can obtain from the IBM IMSL library by using CALL MDNOR (Z,PROB); where Z is a value of the standardized normal distribution  $((X-\mu)/\sigma = (110-120)/5 = -2)$  and PROB is a value we want to get  $(F_2(110))$  [9].

From the standard normal distribution table;  $\Pr(Z \leq -2) = 0.0028$  and by the same way  $F_1(110) = \Pr(X \leq 110 | N(100, 5)) = 0.9772$ . Then,

$$\pi_1(110) = \frac{1 - 0.0228}{1 - 0.9 \times 0.0228} = 0.99767.$$

From Equation 2-14, the total cost of the process is;

$$C(110) = 120 + 0.99767\{(-2.5) - 13.5(0.9772)\} = 104.3.$$

We can find the optimal control limit ( $M_C$ ) which minimizes the total process cost by repeating the above steps.

For the Bayesian Control, the breakeven probability ( $q$ ) can be obtained from Equation 2-25;

$$q = \frac{L - I}{L} = \frac{60 - 15}{60} = 0.75.$$

(d) Now we generate uniform random number by the random number generator. In the IMSL library, GGUBS is available. In this paper, however, the random number generator at the Naval Postgraduate School is used because it is more efficient than GGUBS.

The procedures are as follow: [10]

CALL OVFLOW,

CALL RANDOM (SEED,R,A),

where SEED is a seed for the generation, R is a single array variable that represents random numbers and A is a number that represents how many random numbers we need.

For the numerical example to be discussed later, the random number generated in this step is 0.86565.

(e) In this step, the stochastic process of the state can be decided by using generated random numbers. If the generated random number is less than or equal to the probability of the in-control state, which was calculated in step (b), we decide that the process is in-control. Otherwise, we decide that the process is out-of-control.

In the example, the generated random number (0.86565) is less than the probability of in-control state. Therefore, we decide the process is in-control state.

(f) This step is the same as step (e), except it is for the Bayesian Control. Since the investigating criteria of the Bayesian Control are different from that of Markovian Control, the probability of the in-control state of the Bayesian Control may be different from that of Markovian Control.

Therefore, the separated step was used in the simulation model. In the example, the result is the same as step (e).

Then a cost report has to be generated for the simulation from the distribution of the state which was decided in step (e) and step (f).

(g) This step is applied when the status of the process was decided as in-control state. A cost report is generated from the distribution of the in-control state. The distribution of the in-control state is derived from the non-controllable random deviation around the standard cost ( $\mu_1$ ). Therefore it is appropriate to assume a normal distribution with standard cost as a mean and a certain value derived from past experience as a standard deviation.

A normal random number generator GGNML [9] is available in the IMSL library. But in this paper, a normal random number generator at the Naval Postgraduate School is used.

The procedures for generating normal random number are as follow [10];

CALL OVFLOW

CALL NORMAL (SEED,Z,A);

where SEED is a seed to generate random numbers, Z is a single array variable that represents random number and A is a number that represents the number of random numbers to be generated.

This random number generator generates random numbers from the standard normal distribution (Normal (0,1)). Therefore we

have to transform the standard normal random number into the random number which comes from the distribution we assumed.

$$Z(\text{standard normal}) = \frac{X - \mu}{\sigma};$$

$$X(\text{needed random number}) = \mu + Z\sigma.$$

For example, we assume that a generated random number is 1.0, then the actual random number ( $N_M$ ) from the in-control state ( $N(100,5)$ ) is 105 ( $N_M = 100 + 5 \times 1.0 = 105$ ). Accordingly the cost report from the process is 105 for this period.

Then we have to decide whether or not we should investigate the process based on the cost report. For this, the cost report is compared to the calculated control limit ( $M_C$ ). If the cost report is less than the control limit ( $M_C$ ), we decide that we should not investigate the process. Otherwise, we decide that an investigation is desirable.

In this step, the process was in the in-control state. Therefore if we don't investigate, it doesn't incur any cost ( $C_M = 0$ ), otherwise it incurs an investigation cost ( $C_M = I$ ) in addition to the cost report.

For example, we assume the calculated control limit is 110. Then the cost report (105) is less than the control limit (110). Consequently, we decide not to investigate and the incremental cost is zero.

If we investigate the process, we have to update the probability of an in-control state after investigation, to simulate the next period. Under Condition (1), which assumes

that the probability of an in-control state returns to one after an investigation, the probability of an in-control state is assigned to be one. Under Condition (2), which assumes that the probability of an in-control state doesn't return to one after an investigation, the probability of an in-control state is assigned to be the probability of an in-control state at the end of the current period.

Example: Investigation was conducted:  $P_M' = 1$  (Condition (1));  $P_M' = P_M$  (Condition (2));

Investigation was not conducted:  $P_M' = P_M$ .

(h) This step is applied when the process was decided as out-of-control. A cost report is generated from the distribution of the out-of-control state. The distribution of the out-of-control state is derived from the controllable deviation around a certain expected value ( $\mu_2$ ). It may or may not be a normal distribution. In this paper, normal and gamma distributions were assumed.

If it is assumed that the out-of-control state is a normal distribution, the procedure to obtain a random number is the same as in step (g).

For example, if the distribution of the out-of-control state is  $N(120,5)$  and generated random number from  $N(0,1)$  is  $-1.0$ , then the random number to be used is  $115$  ( $N_M = 120 + 5 \times (-1) = 115$ ). Accordingly the cost report is  $115$  for this period. Then the same procedures as in step (g) are followed.

For example, the cost report (115) is greater than the control limit ( $M_c = 110$ ). Consequently, we decide to investigate

the process. The process is in the out-of-control state.

Therefore, the incremental cost for this period is  $(\mu_2 - \mu_1)$

$$- (I + K): C_M = (120 - 100) - (15 + 5) = 0.$$

The cost of not investigating the process is  $\mu_2 - \mu_1$ , because the process incurs cost  $\mu_2$  instead of  $\mu_1$ . In other words, we lose the savings of  $\mu_2 - \mu_1$ , because we do not investigate the process.

For the gamma distribution, the gamma random number generator GGMAR [9] is available in the IMSL library. The gamma random number generator at the Naval Postgraduate School is also used in this paper.

The procedures are as follow: [11]

CALL OVFLOW

CALL GAMA (G,IX,X,N);

where G is a gamma distribution parameter  $\gamma$ , IX is a seed (integer), X is a single array variable that represents the gamma random numbers and N is a number that represents the number of random numbers to be generated.

The random numbers generated from these procedures are the random numbers from the distribution of  $\Gamma(G,1)$ . But we need random numbers generated from the assumed gamma distribution. Therefore we need transformation.

For example, we assume a gamma distribution with the mean of 120 and the parameter  $\lambda$  of 1. Parameter  $\gamma = 2$  and  $\lambda = 1$  are chosen as a proper shape for the out-of-control state in this paper. Now we assume a random number generated from the  $\Gamma(2,1)$  is 1.50. Then the transformation is as follows:

$$E(X) = \frac{Y}{\lambda} = \frac{2}{1} = 2; \quad \text{Var}(X) = \frac{Y}{\lambda^2} = \frac{2}{1} = 2;$$

$$Y = 85 + 17.5X, \text{ then } E(Y) = 85 + 17.5 \times 2 = 120,$$

$$\text{Var}(Y) = (17.5)^2 \times 2 = 621.5,$$

$$\text{Standard Deviation of } Y = 24.75.$$

Therefore, a needed random number is 116.25 ( $85 + 1.75 \times 1.5$ ).

(i) Now we turn to the Bayesian Control method. This step is applied when the process was decided to be in the in-control state.

The procedures to generate a random number from the distribution of the in-control state are the same as in step (g).

Once a random number (cost report) was generated, then we have to figure out the posterior probability from the information of the cost report. Using Bayes' theorem, the posterior probability can be calculated as follows:

$$f(\theta_j) = f(\theta_j | x) = \frac{f_X(x | \theta_j) f'_n(\theta_j)}{\sum_{j=1}^2 f_X(x | \theta_j) f'_n(\theta_j)};$$

where  $x$  is a cost report,  $f'_n(\theta_1) = g f_{n-1}(\theta_1)$  and  $f'(\theta_2) = 1 - f'_n(\theta_1)$  (Figure 2-3).

We can obtain  $f_X(x | \theta_j)$  from the normal distribution density function.

$$z = \frac{x - \mu}{\sigma}; \text{ where } f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}.$$

By the above two equations, we can calculate  $f_X(x|\theta_j)$  and, in turn, we can calculate  $f(\theta_j)$  for a given cost report  $x$ .

For example, if the generated random number was 105, then:

$$z_1 = \frac{105 - 100}{5} = 1.0, \quad z_2 = \frac{105 - 120}{5} = -3.0,$$

$$f(x|\theta_1) = \frac{1}{\sqrt{2\pi}} e^{-1/2} = 0.242,$$

$$f(x|\theta_2) = \frac{1}{\sqrt{2\pi}} e^{-9/2} = 0.0044,$$

$$f(\theta_1) = \frac{(0.9)(0.242)}{(0.9)(0.242) + (0.1)(0.0044)} = 0.998$$

The revised probability of the in-control state (0.998) is greater than the breakeven probability (0.75). Therefore, we decide not to investigate the process and the incurred incremental cost of the process is zero. The revised probability,  $f(\theta_1)$ , of this period becomes the beginning probability of the next period ( $p_B' = p_{BR} = 0.98$ ). Consequently, the prior probability will be 0.882 ( $0.98 \times 0.9$ ) before generating a cost report for the next period.

If we had investigated the process, the probability of the in-control state at the beginning of the next would have been set to "one" under Condition (1) and to the revised probability of the current period under Condition (2) as the beginning probability of the next period.

For example: If an investigation was conducted;

$$P'_B = 1 \text{ (Condition (1))},$$

$$P'_B = P_{BR} \text{ (Condition (2))}.$$

If an investigation was not conducted;

$$P'_B = P_{BR}.$$

(j) This step is executed when the process is determined to be out-of-control.

The procedures to generate a random number from the distribution of the out-of-control state are the same as in step (h). We also calculate the revised probability by the same procedures as in step (i).

For example, if the generated cost report was 116.25 (step (h)), then;

$$z_1 = \frac{116.25 - 100}{5} = 3.25; \quad z_2 = \frac{116.25 - 120}{5} = -0.75$$

$$f(x|\theta_1) = \frac{1}{\sqrt{2\pi}} e^{-10.56/2} = 0.0020$$

$$f(x|\theta_2) = \frac{1}{\sqrt{2\pi}} e^{-0.56/2} = 0.3011$$

$$f(\theta_1) = \frac{(0.002)(0.9)}{(0.002)(0.9) + (0.1)(.3011)} = 0.056$$

Based on this result, we decide to investigate the process.

The incurred incremental cost of the process is  $(\mu_2 - \mu_1) - (I + K)$ . For example,  $C_B = (\mu_2 - \mu) - (I + K) = (120 - 100) - (15 + 5) = 0$ .

The steps explained above are the steps taken for one iteration of the simulation run. Each iteration puts out two

incurred incremental costs for the process, one for the Markovian Control ( $C_M$ ) and one for the Bayesian Control ( $C_B$ ).

To compare the costs of the two models, a differential cost between the two models ( $C_M - C_B$ ) is calculated for each iteration.

The probability distribution of the calculated differential cost between the two models is unknown. However, according to the central limit theorem; "as the sample size  $n$  increases, the distribution of the mean  $\bar{x}$  of a random sample taken from practically any population approaches a normal distribution" [12], we can assume the sample means ( $\bar{x}_D$ ) of the differential cost are normally distributed.

We can obtain enough sample means to test the significance of the mean of sample means ( $\bar{\bar{x}}_D$ ) and can estimate the variance of the same means ( $s_{\bar{x}_D}^2$ ).

With these values, we can carry out the test of hypotheses as follows:

$$x_D = C_M - C_B,$$

$$\bar{x}_D = \frac{1}{n} \sum_{i=1}^n x_{D_i} \quad (n: \text{number of iterations in each sample})$$

$$\bar{\bar{x}}_D = \frac{1}{m} \sum_{j=1}^m \bar{x}_{D_j} \quad (m: \text{number of samples}),$$

$$s_{\bar{x}_D}^2 = \frac{1}{m-1} \sum_{j=1}^m (\bar{x}_{D_j} - \bar{\bar{x}}_D)^2,$$

Then

$$t_{m-1} = \frac{\bar{\bar{X}}_D - \mu_{X_D}}{S_{\bar{\bar{X}}_D} / \sqrt{m}} ; \quad (\mu_{X_D} : \text{expected value of } X_D).$$

Based on this result, we can test the null hypothesis:  
there is no significant difference between the two models.

$$H_0: \mu_{X_D} = 0 ;$$

$$H_1: \mu_{X_D} \neq 0.$$

If we accept  $H_0$ , we say that there are no significant differences in performance between Markovian Control model and the Bayesian Control model under a given situation. Otherwise, we say that either one of the two methods performs better than the other under a given situation.

In this paper, 100 iterations for each sample and 50 samples are implemented.

APPENDIX B  
COMPUTER PROGRAMS

```

INTEGER IX,IY
REAL UI,U0,SI,SO,G,CI,CK,A,B,PMP,PMT,PBP,PBT,N,M,PG,
1XCON,FXI,FXO,PI,DEFF,DEFT,DEBAR,DSQS,DSQ,DTBAR,DEFTT,
4DTVAR,CPM,CPB,XM,XB,CRITE,CLOSS,PBR,STD1,STD0,XBTT,
8XMTT,XMT,XBT,XMBAR,XBBAR,XMTBR,XBTBR,DMQ,DBQ,MQ,BQ,
9DMVAR,DBVAR
DIMENSION N(5000),M(5000),XM(50,100),XB(50,100),
2CPM(50,100),CPB(50,100),DEFF(50,100),DEFT(50),
5DEBAR(50),DSQ(50),PBR(50,100),SCORE(50),XMT(50),
6XB(50),XMBAR(50),XBBAR(50),MQ(50),BQ(50)
READ(5,10) UI,U0,SI,SO,G,CI,CK,IX,IY,CLOSS
PI = 3.14286
A = CI + (1-G)*CK-G*(U0-UI)
B = G*CI
PMT = 1
PBT = 1
XCON=CRITE(UI,U0,SI,SO,PI,G,A,B)
Pj=(CLOSS-CI)/CLOSS
CALL OVFLOW
CALL RANDOM(IX,N,5000)
CALL NORMAL(IY,M,5000)
DO 20 I=1,50
  DO 30 J=1,100
    K=J+(I-1)*100
    PMT=PMT*G
    PBP=PBT*G
    IF(N(K).GE.PMP) GO TO 40
    XM(I,J)=M(K)*SI+UI
    IF(XM(I,J).GE.XCON) GO TO 50
    CPM(I,J)=0
    PMT=PMP
    GO TO 70
  50      CPM(I,J)=CI
    PMT=1 (Condition 0))PMT=PMP (Condition 2))
    GO TO 70
  40      XM(I,J)=M(K)*SO+U0
    IF(XM(I,J).GE.XCON) GO TO 60
    CPM(I,J)=UC-UI
    PMT=PMP
    GO TO 70
  60      CPM(I,J)=(CI+CK)-(U0-UI)
    PMT=1
  70      IF(N(K).GE.PBP) GO TO 80
    XB(I,J)=M(K)*SI+UI
    STD1=(XB(I,J)-UI)/SI
    STD0=(XB(I,J)-U0)/SO
    FXI=EXP(-(STD1**2/2))/(SQRT(2*PI))
    FXO=EXP(-(STD0**2/2))/(SQRT(2*PI))
    PBR(I,J)=(FXI*PBP)/(FXI*PBP+FXO*(1-PBP))
    IF(PBR(I,J).LE.PQ) GO TO 90
    CPB(I,J)=0
    PBT=PBR(I,J)
    GO TO 110
  90      CPB(I,J)=CI
    PBT=1 (Condition(1))PBT=PBK(I,J) (Condition(2))
    GO TO 110
  80      XB(I,J)=M(K)*SO+U0
    STD1=(XB(I,J)-UI)/SI
    STD0=(XB(I,J)-U0)/SO
    FXI=EXP(-(STD1**2/2))/(SQRT(2*PI))
    FXO=EXP(-(STD0**2/2))/(SQRT(2*PI))
    PBR(I,J)=(FXI*PBP)/(FXI*PBP+FXO*(1-PBP))
    IF(PBR(I,J).LE.PQ) GO TO 100
    CPB(I,J)=UC-UI
    PBT=PBR(I,J)
    GO TO 110
  100     CPB(I,J)=(CI+CK)-(U0-UI)

```

```

      PBT=1
110      DEFF(I,J)=CPM(I,J)-CPB(I,J)
30      CONTINUE
20      CONTINUE
      WRITE(6,230)UI,U0,SI,SO,G,CI,CK,PQ
      DEFTT=0
      XMTT=0
      X3TT=0
      DO 150 I=1,50
      WRITE(6,250) I
      DEFT(I)=0
      XMT(I)=0
      XBT(I)=0
      DO 160 J=1,100,2
      DEFT(I)=DEFT(I)+DEFF(I,J)+DEFF(I,J+1)
      XMT(I)=XMT(I)+XM(I,J)+XM(I,J+1)
      XBT(I)=XBT(I)+XB(I,J)+XB(I,J+1)
      DEBAR(I)=DEFT(I)/100
      XMBAR(I)=XMT(I)/100
      XBBAR(I)=XBT(I)/100
160      CONTINUE
      WRITE(6,210) DEBAR(I),XMBAR(I),XBBAR(I)
      DEFTT=DEFTT+DEBAR(I)
      XMTT=XMTT+XMBAR(I)
      XBTT=XBTT+XBBAR(I)
150      CONTINUE
      DTBAR=DEFTT/50
      XMTBR=XMTT/50
      XBTBR=XBTT/50
      DSQS=0
      DMQ=0
      DBQ=0
      DO 170 I=1,50
      DSQ(I)=(DEBAR(I)-DTBAR)**2
      MQ(I)=(XMBAR(I)-XMTBR)**2
      BQ(I)=(XBBAR(I)-XBTBR)**2
      DSQS=DSQS+DSQ(I)
      DMQ=DMQ+MQ(I)
      DBQ=DBQ+BQ(I)
170      CONTINUE
      DTVAR=DSQS/49
      D1VAR=DMQ/49
      DBVAR=DBQ/49
      WRITE(6,220) DTBAR,DTVAR,XMTBR,DMVAR,XBTBR,DBVAR
      CALL HISTF(DEBAR,50,15)
      CALL NORMPL(DEBAR,SCORE,50,3)
      STOP
10      FORMAT(7F6.2,2I7,F6.2)
210     FORMAT(1,3(2XF20.5))
220     FORMAT(1,6F18.5)
230     FORMAT('0',3X,7F10.2,13X,F8.5)
250     FORMAT(1,10X,'REPLICATION = ',I4)
      END
      FUNCTION CRITE(UI,UG,SI,SO,PI,G,A,B)
      REAL UI,U0,SI,SO,PI,XMIN,XMAX,YIN,YOUT,FXN,FXA,CMIN,
      3CMAX,EQY,EQW,WIN,WOUT,A,B,GXI,GXA,ERR,DX
      DX=(U0-UI)/20
      XMAX=UI
400      XMIN=XMAX
      XMAX=XMIN+DX
      YIN=(XMIN-UI)/SI
      YOUT=(XMIN-U0)/SO
      CALL MDNOR(YIN,FXN)
      CALL MDNOR(YOUT,FXA)
      EQY=(1-FXA)/(1-G*FXA)
      CMIN=U0+EQY*(A-B*FXN)
      WIN=(XMAX-UI)/SI

```

```
WOUT=(XMAX-U0)/SO
CALL MONOR(WIN,GXI)
CALL MONOR(WOUT,GXA)
EQW=(1-GXA)/(1-G*GXA)
CMAX=U0+EQW*(A-B*GXI)
ERR=ABS(CMAX-CMIN)
IF(ERR.LE.0.001) GO TO 300
IF(CMAX.LT.CMIN) GO TO 400
DX=-{DX/10}
GO TO 400
300 CRITE=(XMIN+XMAX)/2
WRITE(6,240) CRITE
240 FFORMAT(//,74X,F9.5)
RETURN
END
```

```

INTEGER IX,IY
REAL UI,U0,SI,SD,G,CI,CK,A,B,PMP,PMT,PBP,PBT,N,M,PQ,
1XCON,FXI,FXO,PI,DEFF,DEFT,DEBAR,DSQS,DSQ,DTBAR,DEFTI,
4DTVAR,CPM,CPB,XM,XB,CRITE,CLOSS,PBR,STD1,STD0,R,XGAM,
8XBT,XMT,XBT,XMBAR,XBBAR,XMTBR,XBTBR,DMQ,DBQ,MQ,
9BQ,DMVAR,DBVAR
DIMENSION N(5000),M(5000),XM(50,100),XB(50,100),
2CPM(50,100),CPB(50,100),DEFF(50,100),DEFT(50),
5DEBAR(50),DSQ(50),PBR(50,100),GAM(5000),SCORE(50),
6XMT(50),XBT(50),XMBAR(50),XBBAR(50),MQ(50),BQ(50)
READ(5,10) UI,U0,SI,SD,G,CI,CK,IX,IY,CLOSS,R
CALL OVFLOW
CALL RANDOM(IX,N,5000)
CALL NORMAL(IY,M,5000)
CALL GAMA(R,IY,GAM,5000)
PI = 3.14286
A = CI + (1-G)*CK-G*(U0-UI)
B = G*CI
PMT = 1
PBT = 1
XCON=CRITE(UI,U0,SI,SD,R ,G,A,B)
PQ=(CLOSS-CI)/CLOSS
DIV=GAMMA(R)
DO 20 I=1,50
  DO 30 J=1,100
    K=J+(I-1)*100
    PMP=PMT*G
    PBP=PBT*G
    IF(N(K).GE.PMP) GO TO 40
    XM(I,J)=M(K)*SI+UI
    IF (XM(I,J).GE.XCON) GO TO 50
    CPM(I,J)=0
    PMT=PMP
    GO TO 70
  50  CPM(I,J)=CI
    PMT=PMP (Condition(2))PMT=1(Condition(1))
    GO TO 70
  40  XM(I,J)=85+GAM(K)*17.5
    IF(XM(I,J).GE.XCON) GO TO 60
    CPM(I,J)=UC-UI
    PMT=PMP
    GO TO 70
  60  CPM(I,J)=(CI+CK)-(U0-UI)
    PMT=1
  70  IF(N(K).GE.PBP) GO TO 80
    XB(I,J)=M(K)*SI+UI
    STD1=(XB(I,J)-UI)/SI
    XGAM=(XB(I,J)-85)/17.5
    IF(XGAM.LE.0) GO TO 65
    FXO=EXP(-XGAM)*(XGAM***(R-1))/DIV
    GO TO 66
  65  FXO=0
  66  FXI=EXP(-(STD1**2/2))/(SQRT(2*PI))
    PBR(I,J)=(FXI*PBP)/(FXI*PBP+FXO*(1-PBP))
    IF(PBR(I,J).LE.PQ) GO TO 90
    CPB(I,J)=0
    PBT=PBR(I,J)
    GO TO 110
  90  CPB(I,J)=CI
    PBT=PBR(I,J) (Condition(2))PBT=1(Condition(1))
    GO TO 110
  80  XB(I,J)=85+GAM(K)*17.5
    STD1=(XB(I,J)-UI)/SI
    FXI=EXP(-(STD1**2/2))/(SQRT(2*PI))
    FXO=EXP(-GAM(K)*(GAM(K)***(R-1))/DIV
    PBR(I,J)=(FXI*PBP)/(FXI*PBP+FXO*(1-PBP))
    IF(PBR(I,J).LE.PQ) GO TO 100

```

```

CPB(I,J)=UC-UI
PBT=PBR(I,J)
GO TO 110
100      CPB(I,J)=(CI+CK)-(UO-UI)
PBT=1
110      DEFF(I,J)=CPM(I,J)-CPB(I,J)
30      CONTINUE
20      CONTINUE
      WRITE(6,230)UI,UO,SI,SO,G,CI,CK,PQ
      DEFTT=0
      XMTT=0
      XBTT=0
      DO 150 I=1,50
      WRITE(6,250) I
      DEFT(I)=0
      XMT(I)=0
      XBT(I)=0
      DO 160 J=1,100,2
      DEFT(I)=DEFT(I)+DEFF(I,J)+DEFF(I,J+1)
      XMT(I)=XMT(I)+XM(I,J)+XM(I,J+1)
      XBT(I)=XBT(I)+XB(I,J)+XB(I,J+1)
160      CONTINUE
      DEBAR(I)=DEFT(I)/100
      XMBAR(I)=XMT(I)/100
      XBBAR(I)=XBT(I)/100
      WRITE(6,210) DEBAR(I),XMBAR(I),XBBAR(I)
      DEFTT=DEFTT+DEBAR(I)
      XMTT=XMTT+XMBAR(I)
      XBTT=XBTT+XBBAR(I)
150      CONTINUE
      DTBAR=DEFTT/50
      XMTBR=XMTT/50
      XBTBR=XBTT/50
      DSQS=0
      DMQ=0
      DBQ=0
      DO 170 I=1,50
      DSQ(I)=(DEBAR(I)-DTBAR)**2
      MQ(I)=(XMBAR(I)-XMTBR)**2
      BQ(I)=(XBBAR(I)-XBTBR)**2
      DSQS=DSQS+DSQ(I)
      DMQ=DMQ+MQ(I)
      DBQ=DBQ+BQ(I)
170      CONTINUE
      DTVAR=DSQS/49
      DMVAR=DMQ/49
      DBVAR=DBQ/49
      WRITE(6,220) DTBAR,CTVAR,XMTBR,DMVAR,XBTBR,DBVAR
      CALL HISTF(DEBAR,50,15)
      CALL NORMPL(DEBAR,SCORE,50,3)
      STOP
10      FORMAT(7F6.2,2I7,2F6.2)
210     FORMAT(/,3(2X,F20.5))
220     FORMAT(//,6F18.5)
230     FORMAT('0',3X,7F10.2,13X,F8.5)
250     FORMAT(//,10X,'REPLICATION = ',I4)
      END
      FUNCTION CRITE(UI,UO,SI,SO,P1,XMIN,XMAX,YIN,YOUT,FXN,FXA,CMIN,
3CMAX,EQY,EQW,WIN,WOUT,A,B,GXI,GXA,ERR,DX,R
      DX=(UO-UI)/20
      XMAX=UI
      XMIN=XMAX
      XMAX=XMIN+DX
      YIN=(XMIN-UI)/SI
      YOUT=(XMIN-85)/17.5
      IF(YOUT.LE.0) GO TO 401

```

```

        CALL MDGAM(YOUT,R,FXA,129)
401  GO TO 402
402  FXA=0
        CALL MDNOR(YIN,FXN)
        EQY=(1-FXA)/(1-G*FXA)
        CMIN=U0+EQY*(A-B*FXN)
        WIN=(XMAX-UI)/SI
        WOUT=(XMAX-85)/17.5
        IF (WOUT.LE.0) GO TO 403
        CALL MDGAM(WOUT,R,GXA,129)
        GO TO 404
403  GXA=0
404  CALL MDNOR(WIN,GXI)
        EQW=(1-GXA)/(1-G*GXA)
        CMAX=U0+EQW*(A-B*GXI)
        ERR=ABS(CMAX-CMIN)
        IF (ERR.LE.0.001) GO TO 300
        IF (CMAX.LT.CMIN) GO TO 400
        DX=-(DX/10)
        GO TO 400
300  CRITE=(XMIN+XMAX)/2
        WRITE(6,240) CRITE
240  FORMAT(76X,F9.5)
        RETURN
        END

```

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